Infinite-state Liveness Checking with RLive

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- **Extend RLive to infinite-state systems**

RLive-inf Setup

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- **Falsify** φ by finding counterexample lasso $\pi = \pi_{prefix} \cdot \pi_{loop}^{\omega}$
- **Prove** φ by showing $\neg q$ is only finitely reachable

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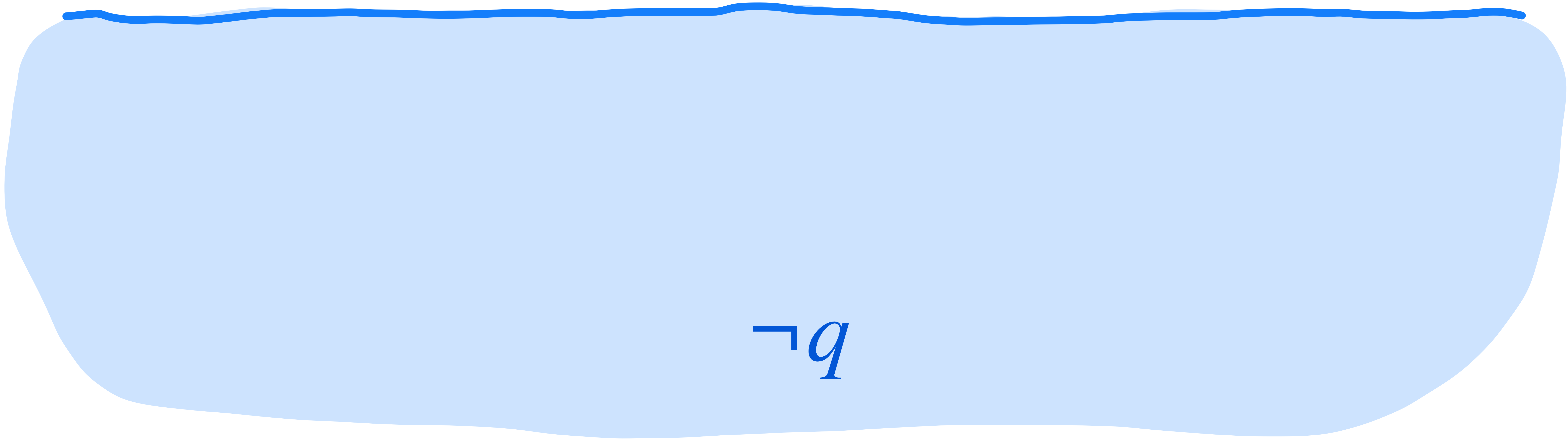

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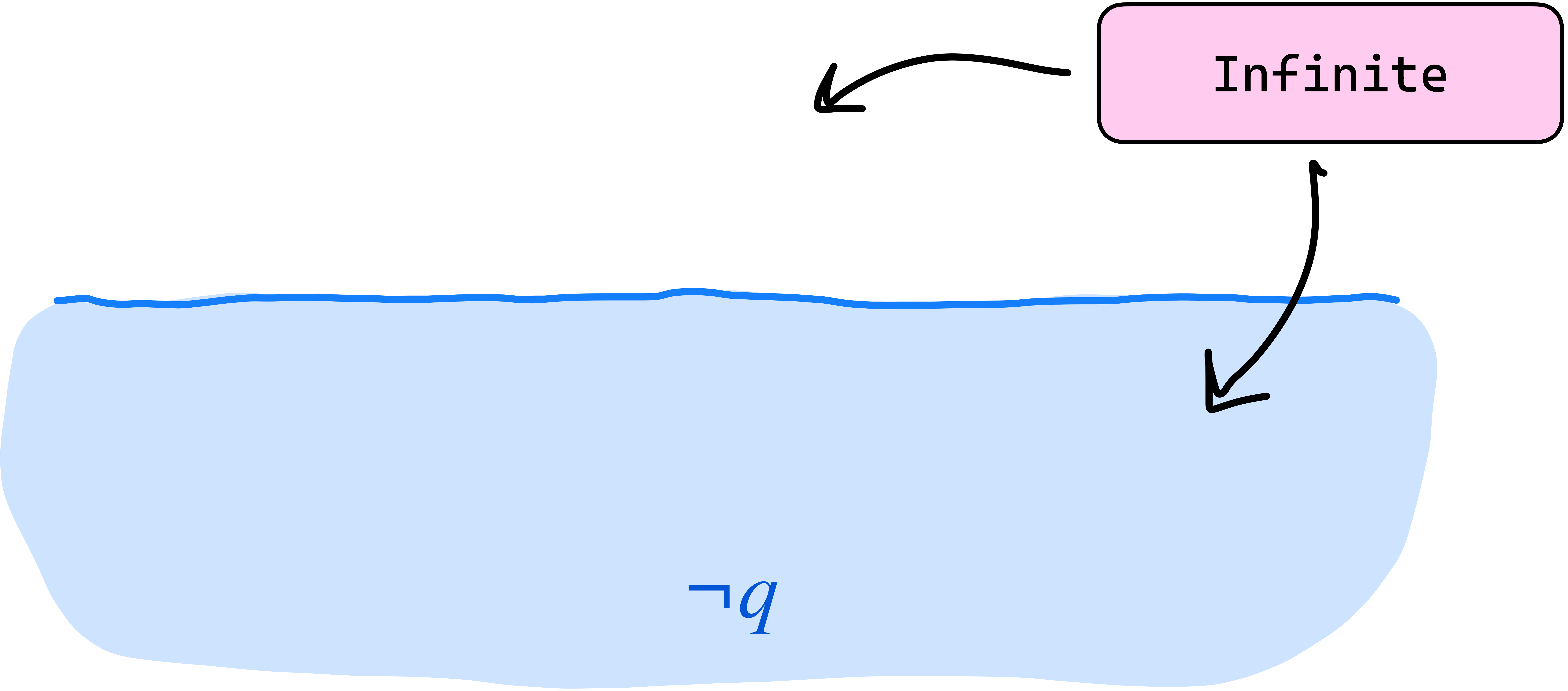

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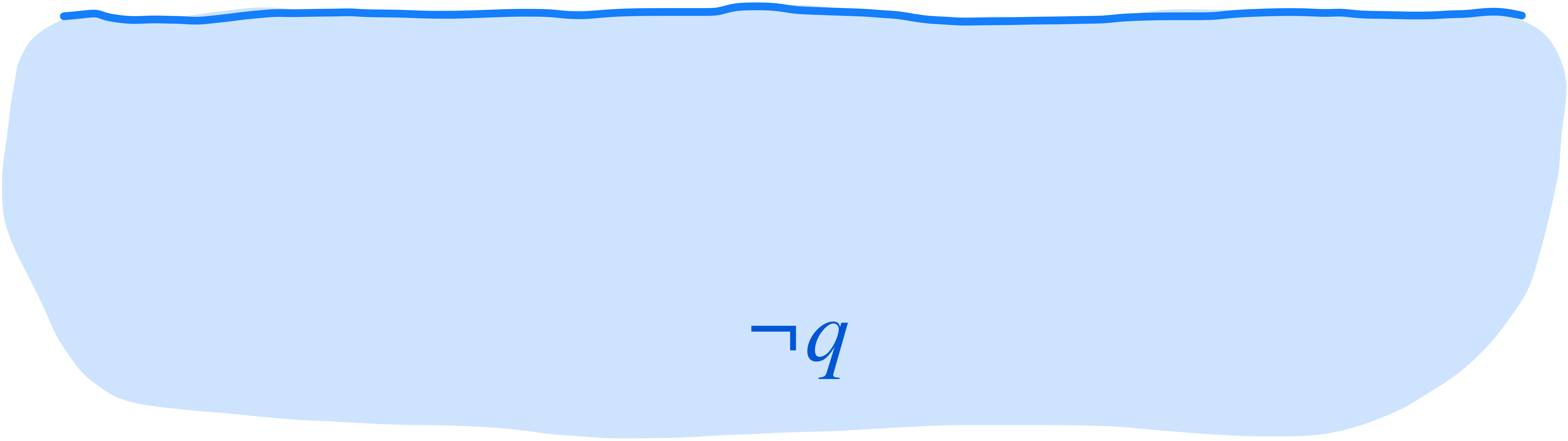
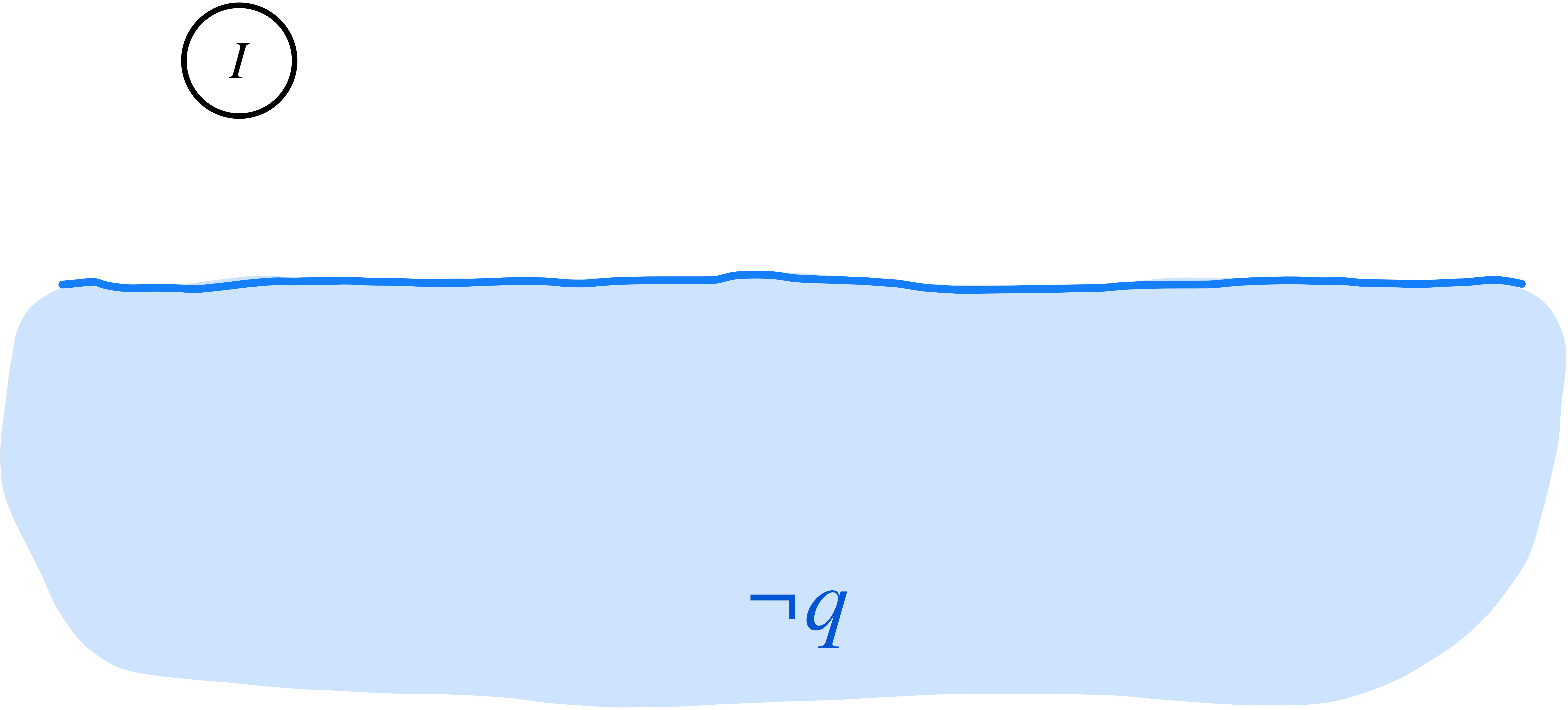


$$S = \langle X, I(X), T(X, X') \rangle$$

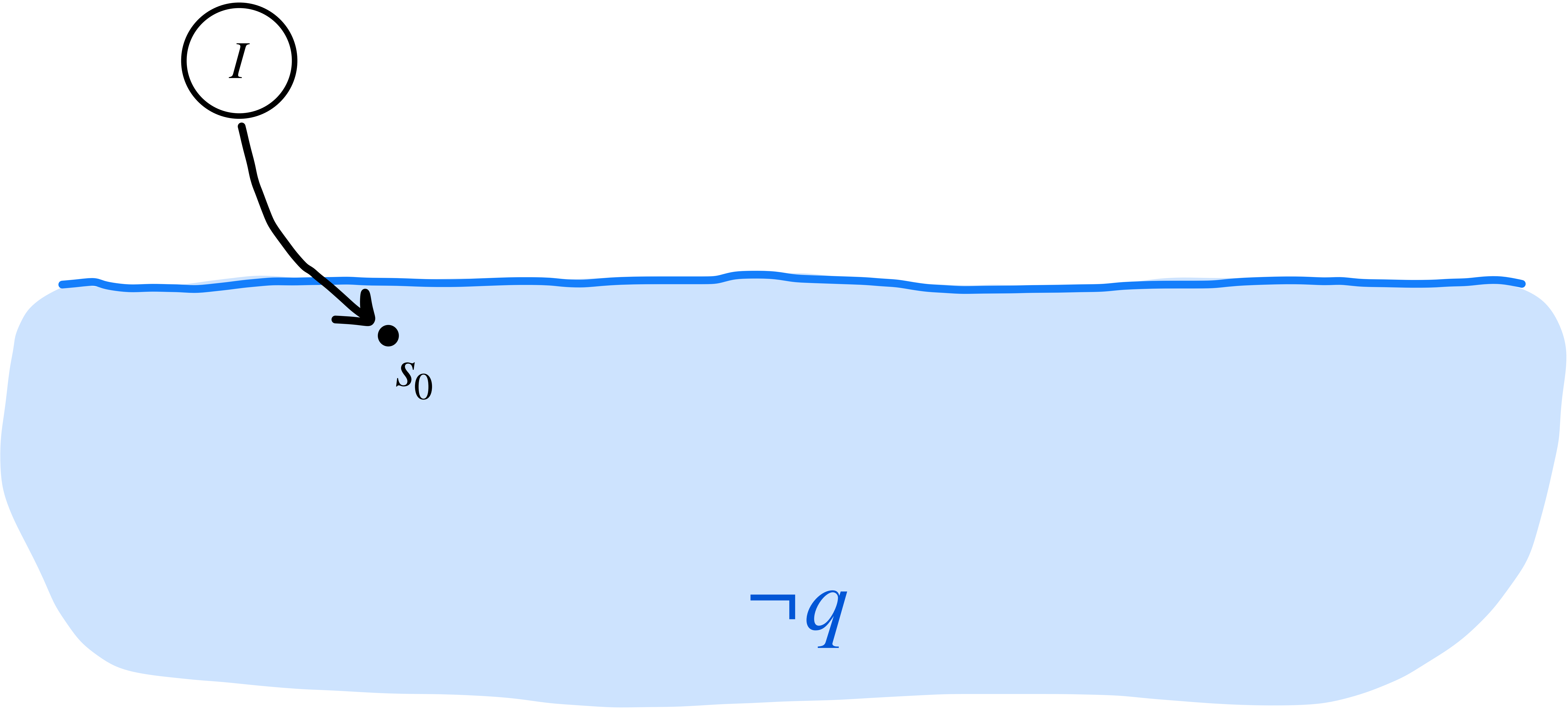
Search Stack:

$$\varphi = \Diamond \Box q$$



$S = \langle X, I(X), T(X, X') \rangle$ **Search Stack:** s_0

$\varphi = \Diamond \Box q$



s_0

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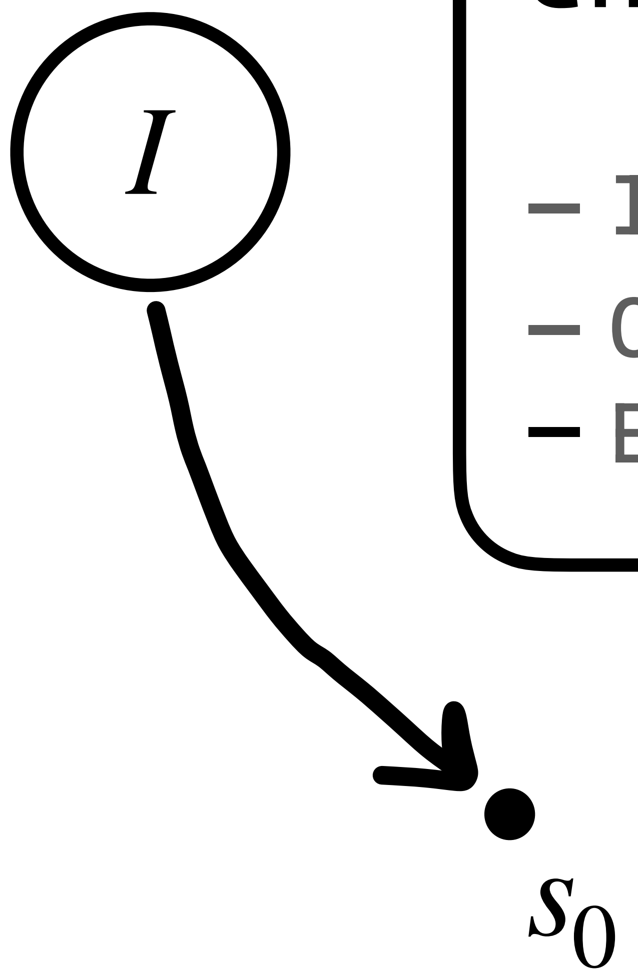

check-inv(I, T, q)

- If $\neg q$ is reachable, returns counterexample with final state s_0
- Otherwise, returns inductive invariant C
- Example implementations: ic3ia, pdkind, etc.

$\neg q$

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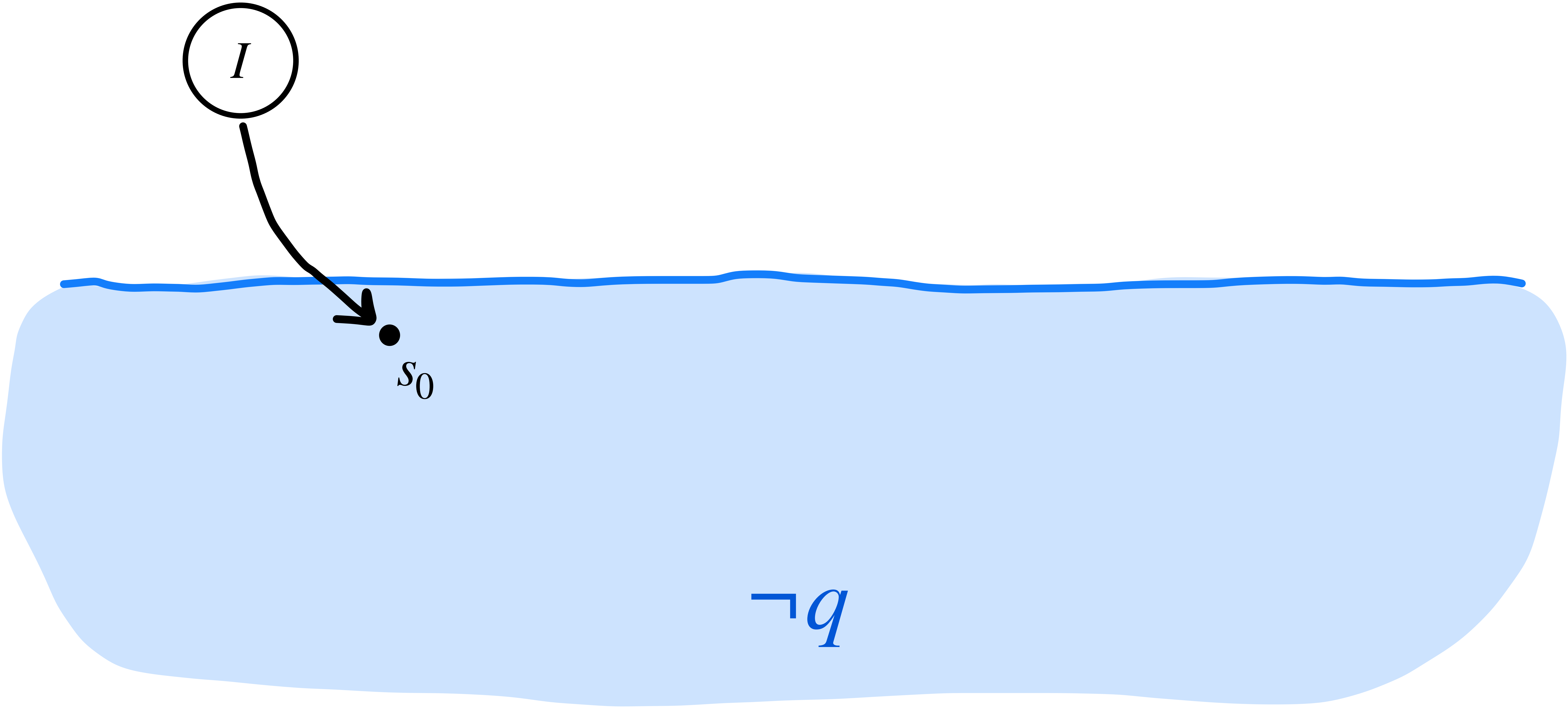

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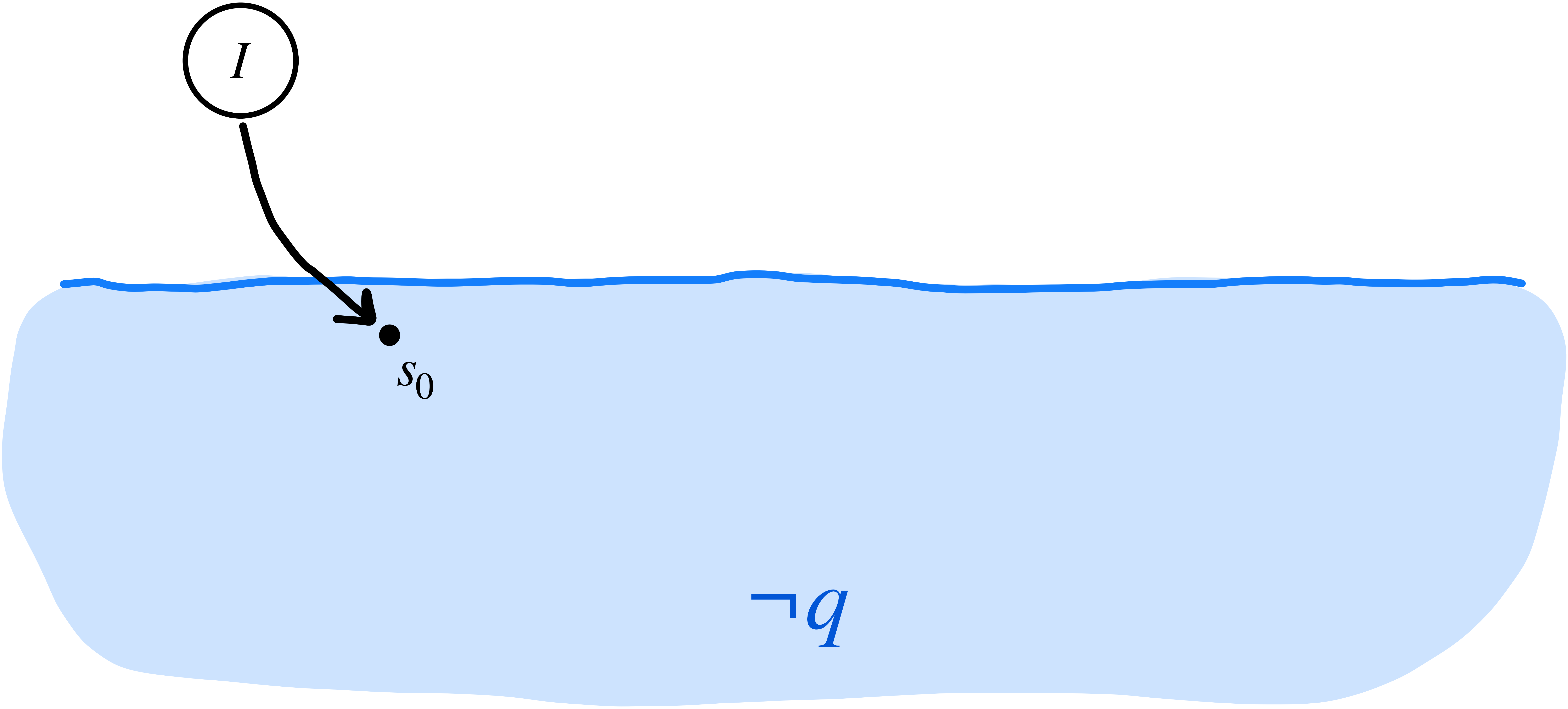

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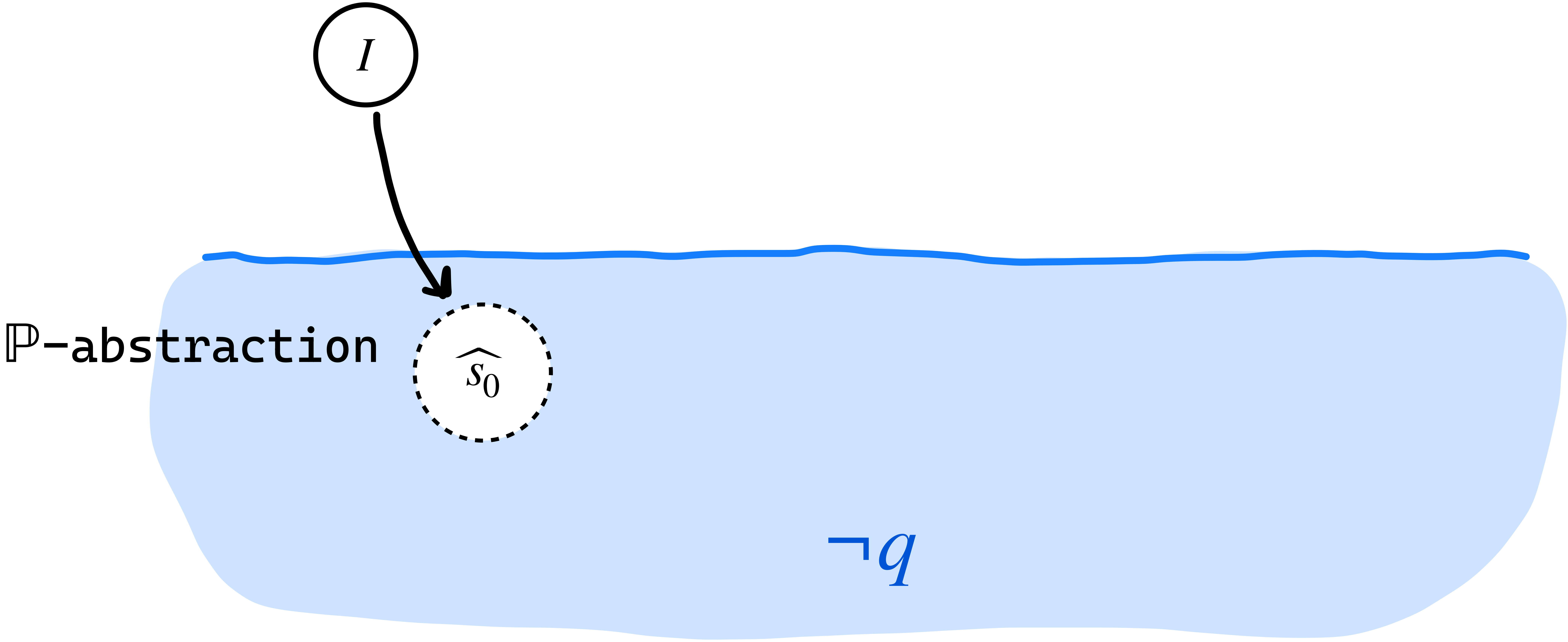

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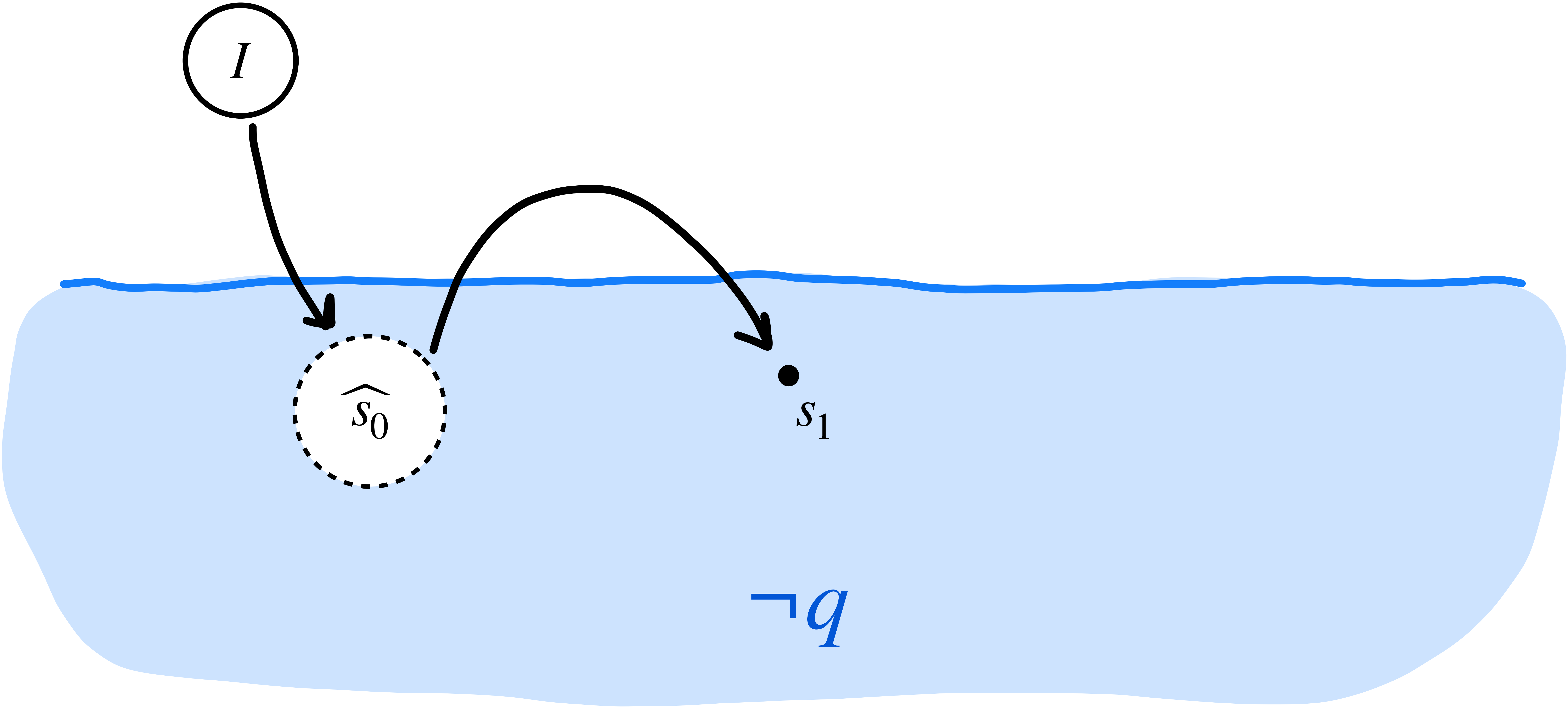

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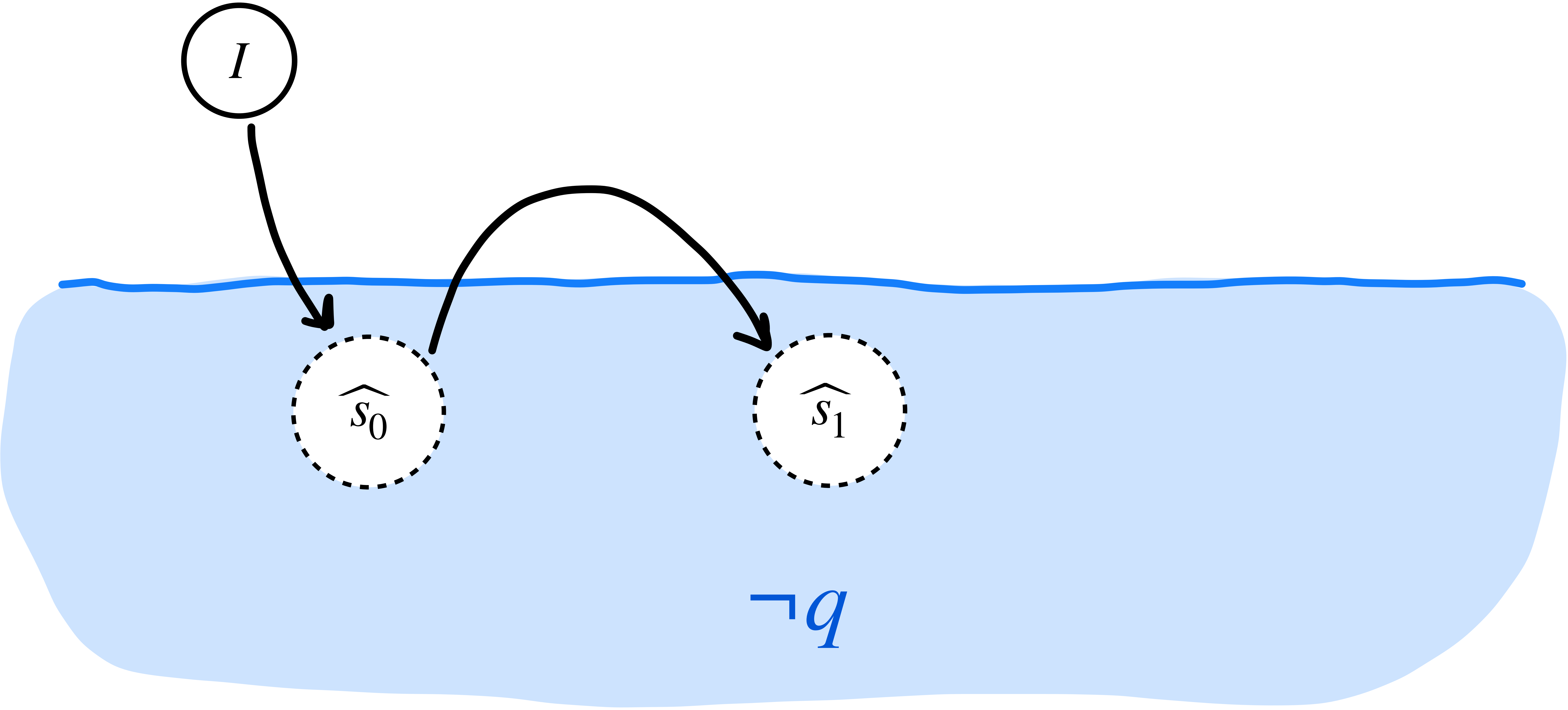

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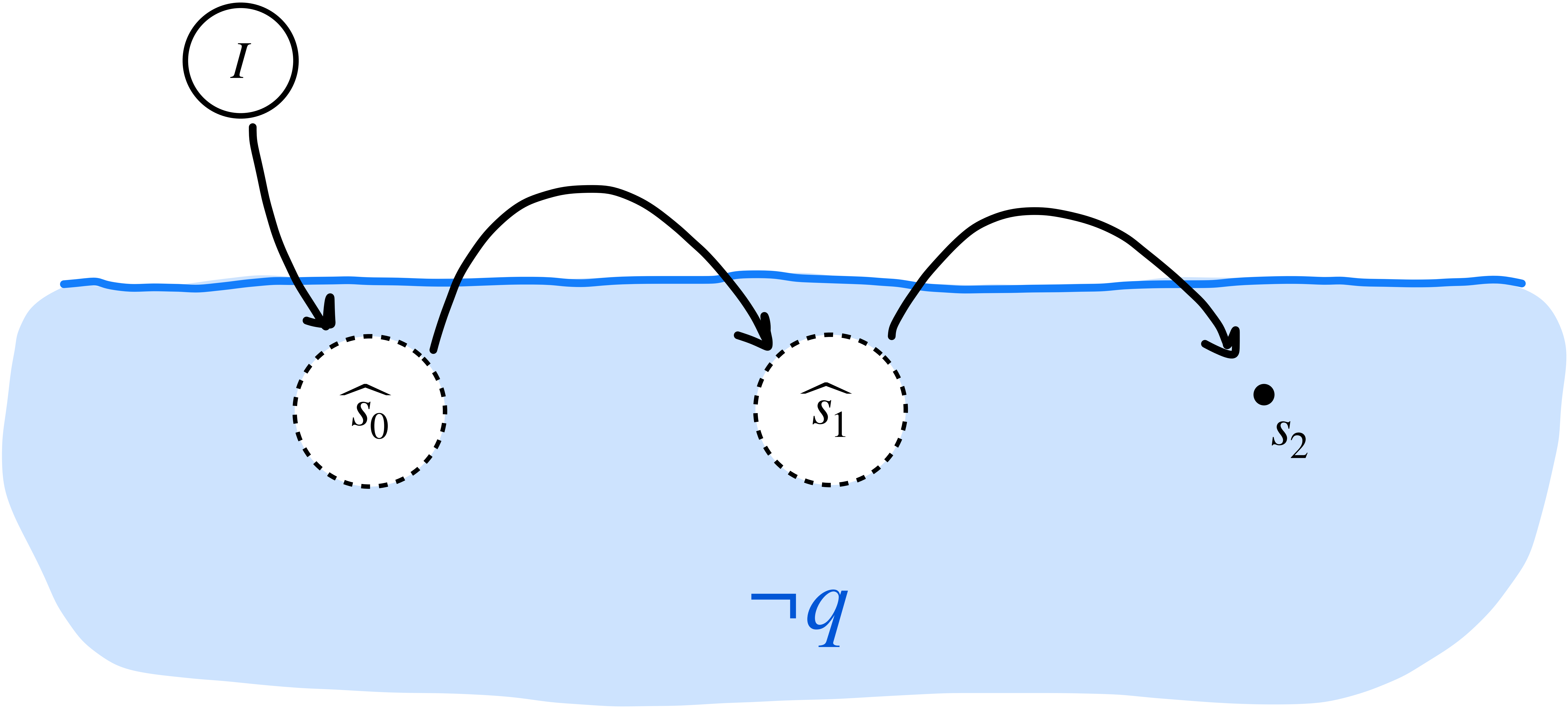

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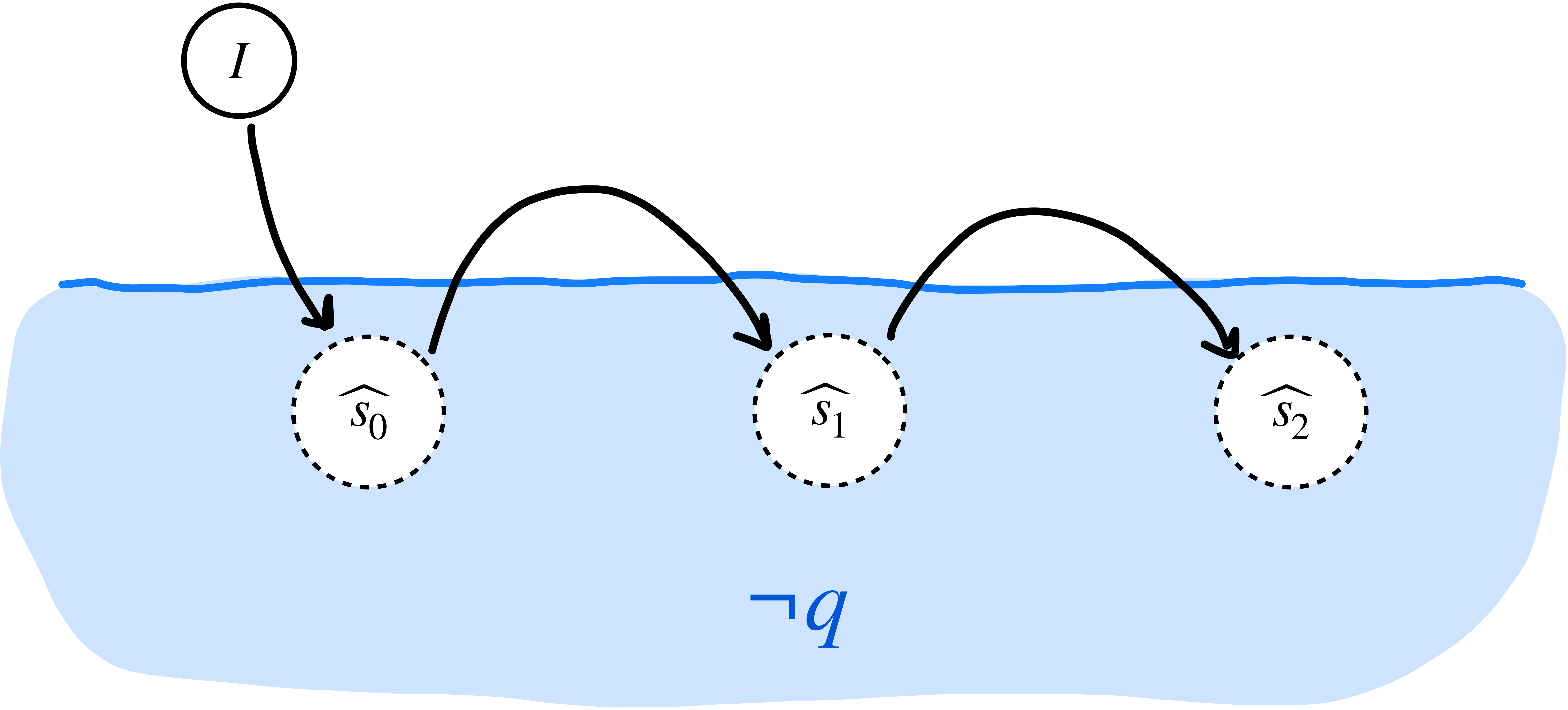

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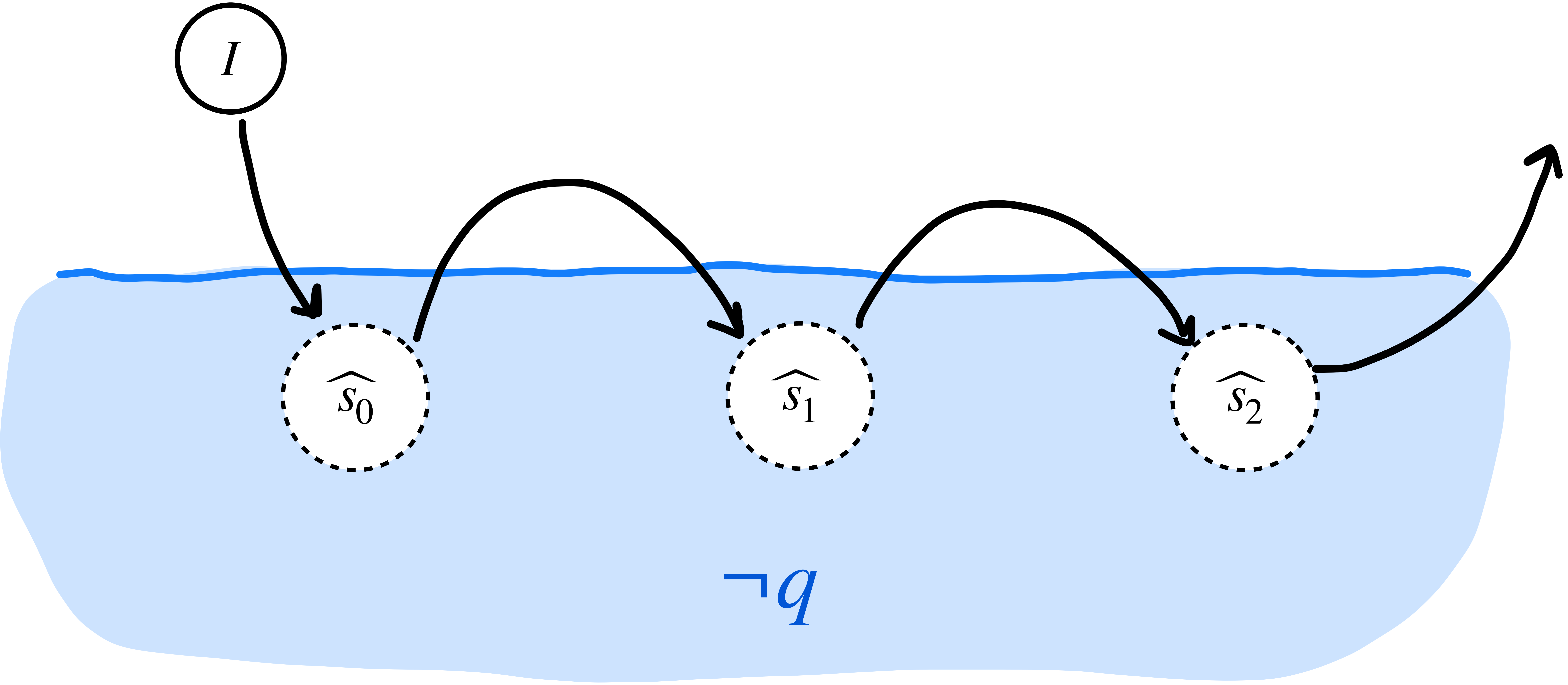

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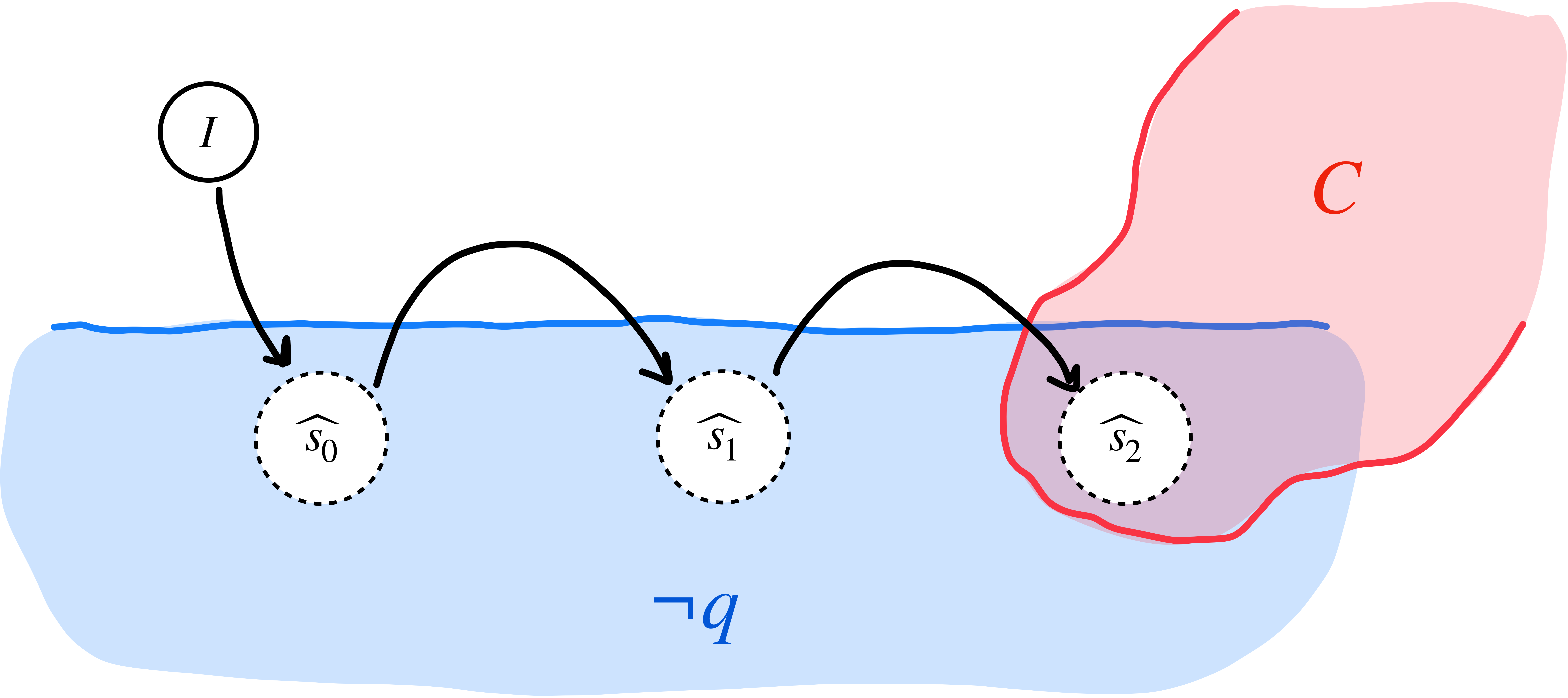

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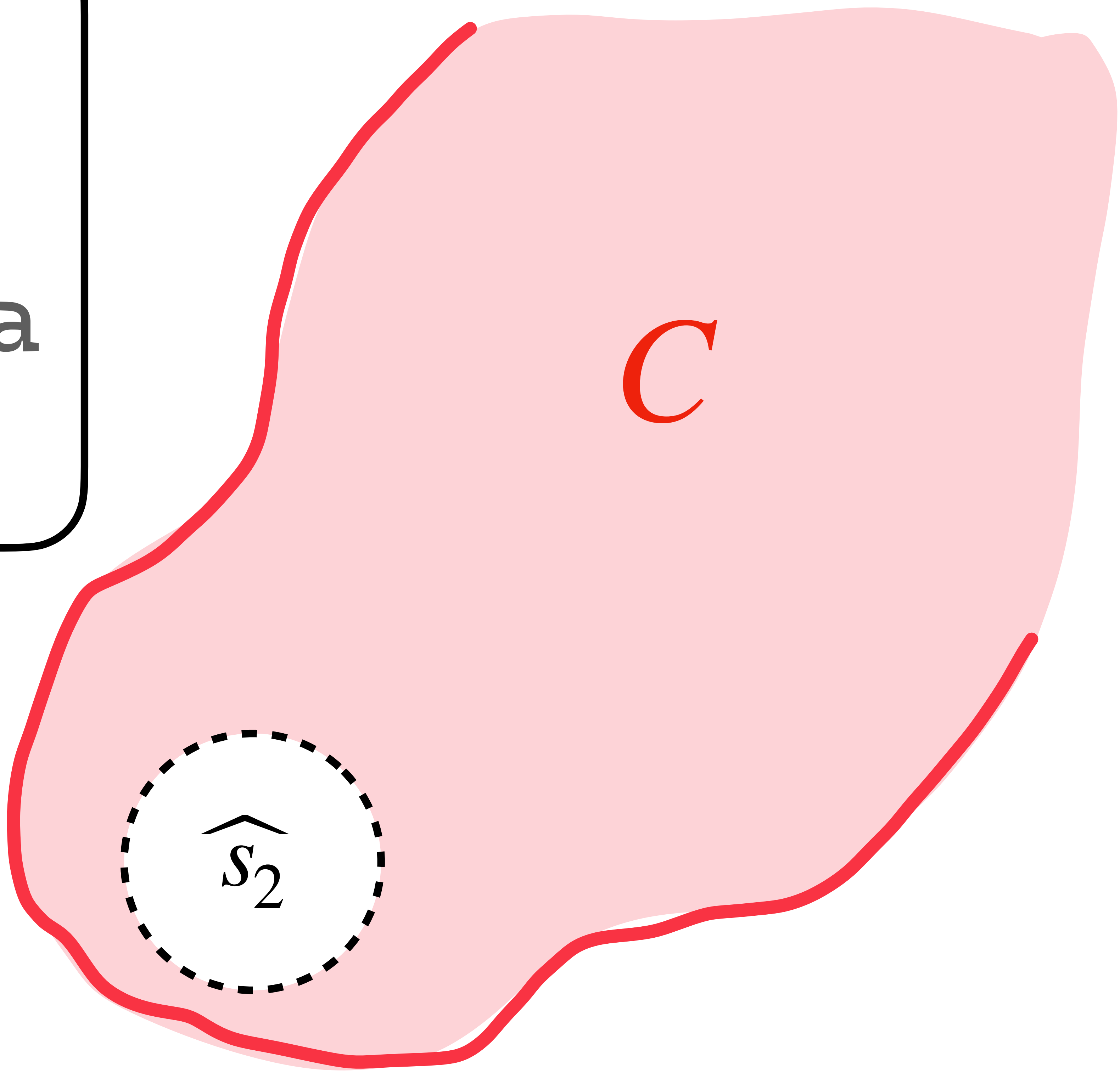
$\varphi = \Diamond\Box q$ $\quad$ $\mathbb{P} = \{p_0, p_1, \ldots\}$

(Shoals) $\quad C =$ `get-invariant()`

- Set of states that can reach $\neg q$ only a **finite** number of times



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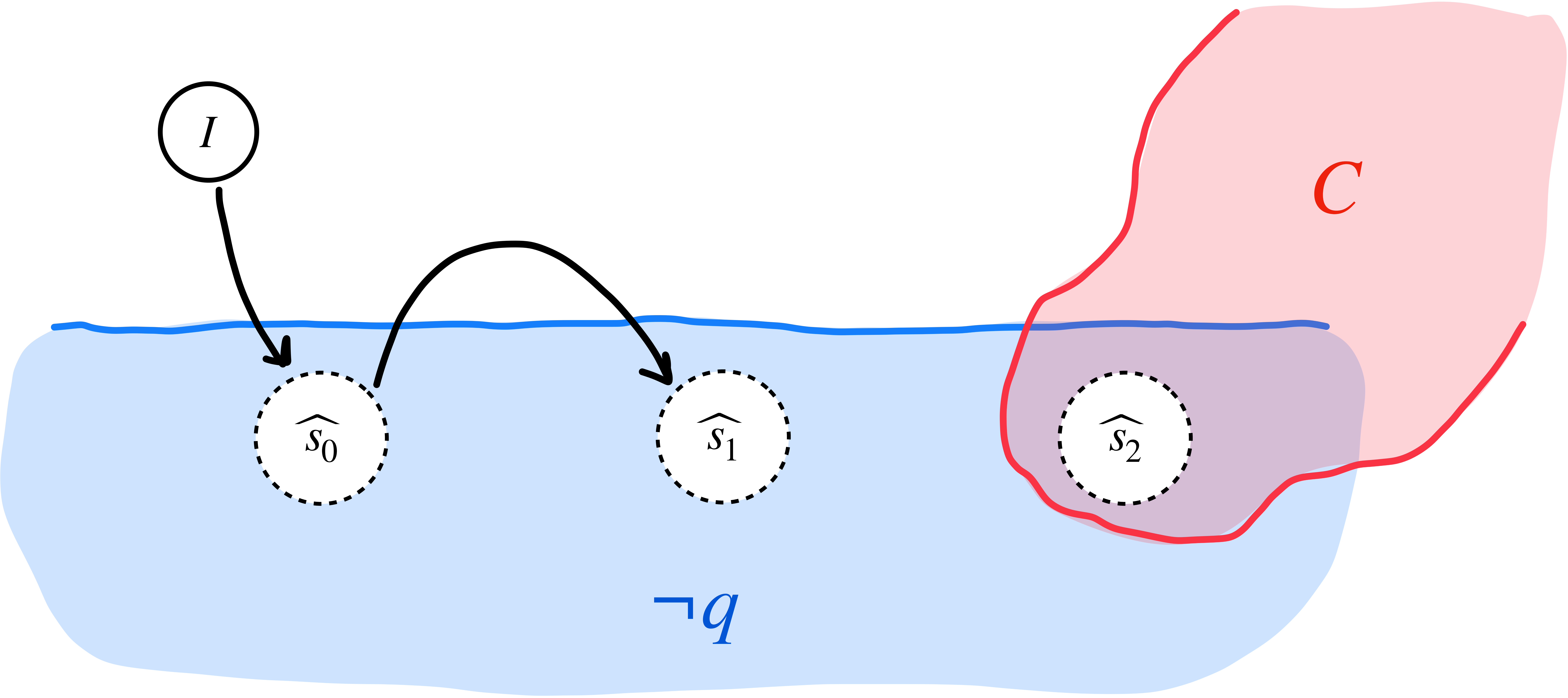

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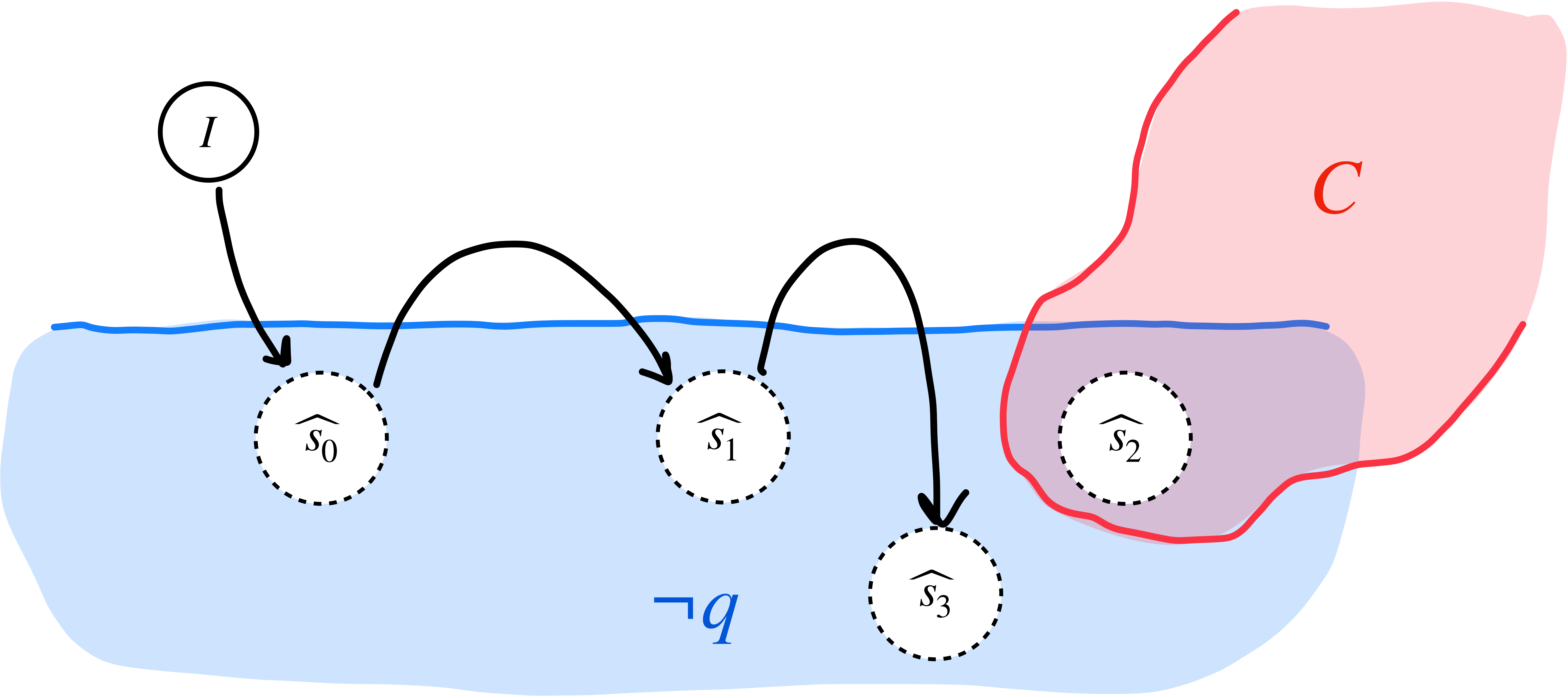

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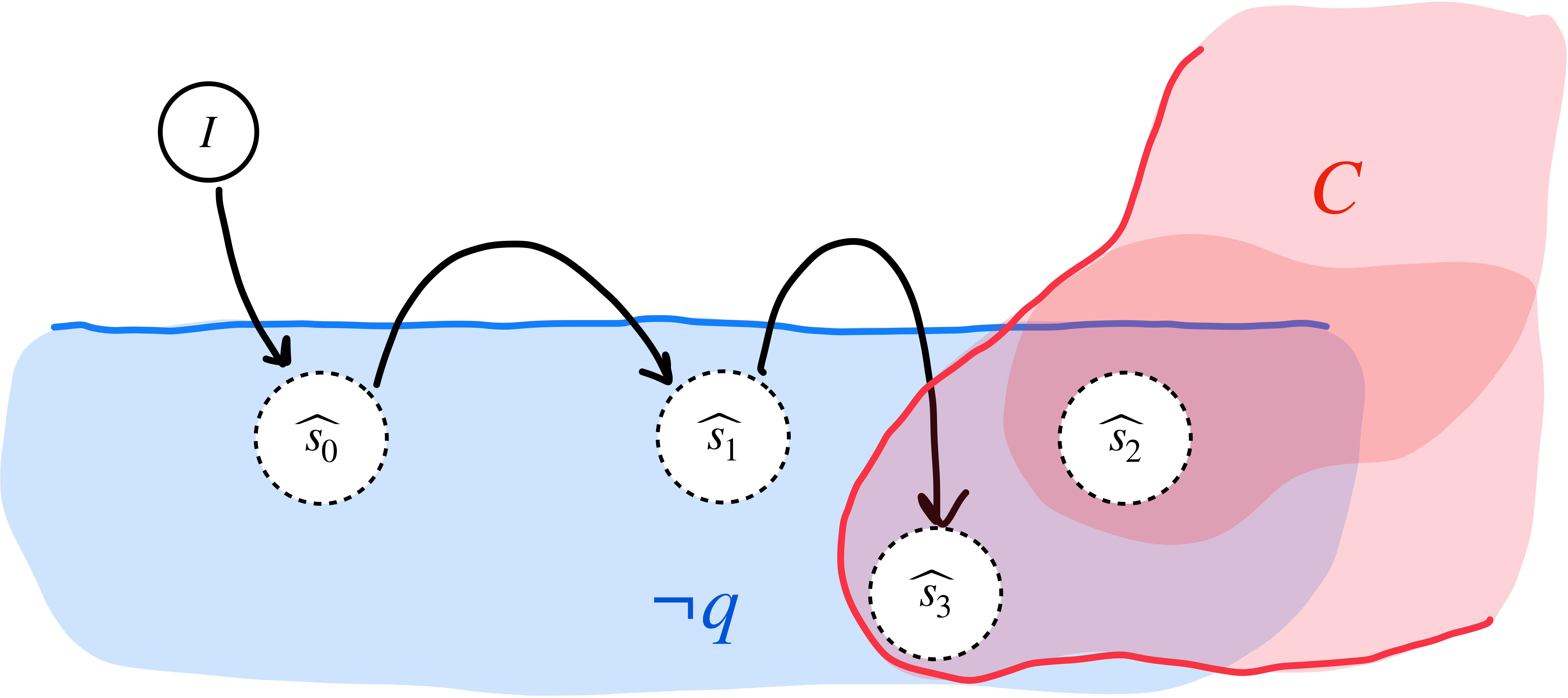

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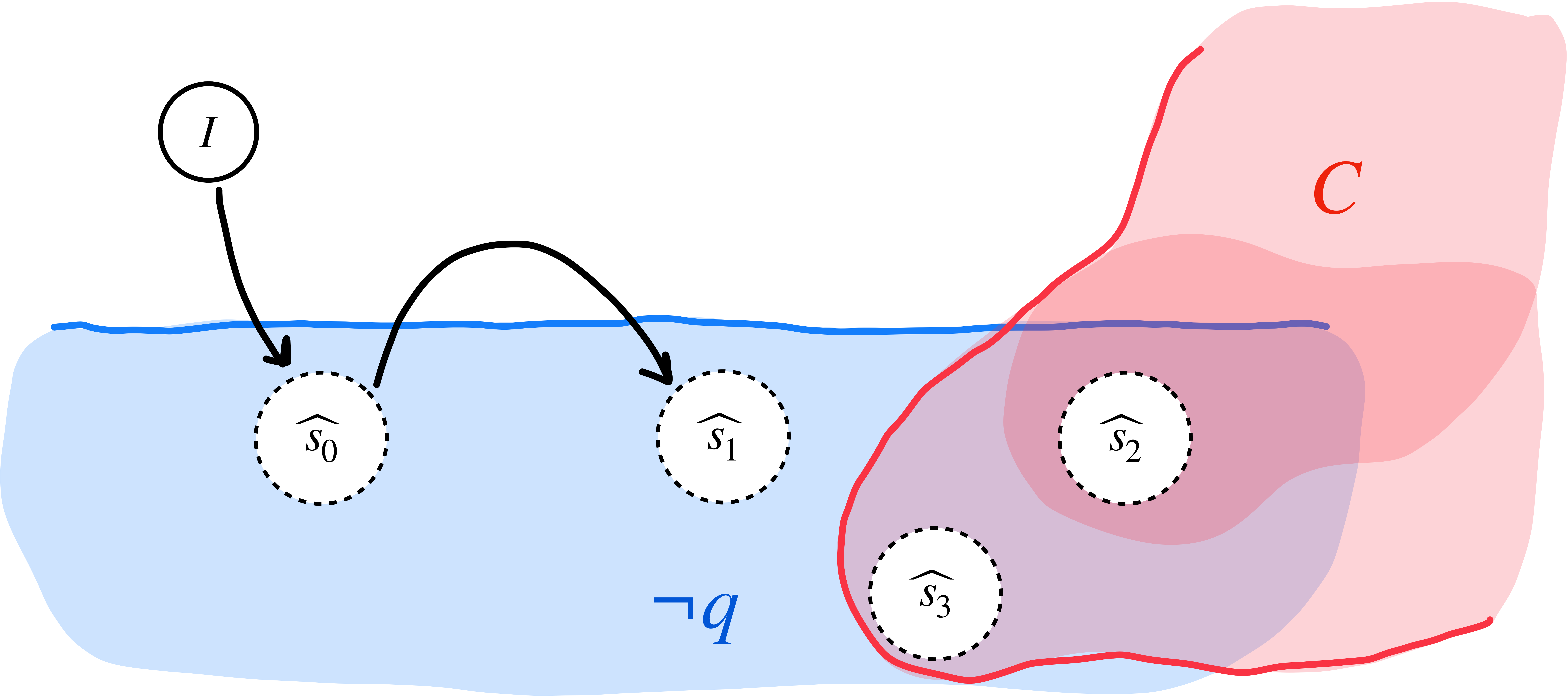

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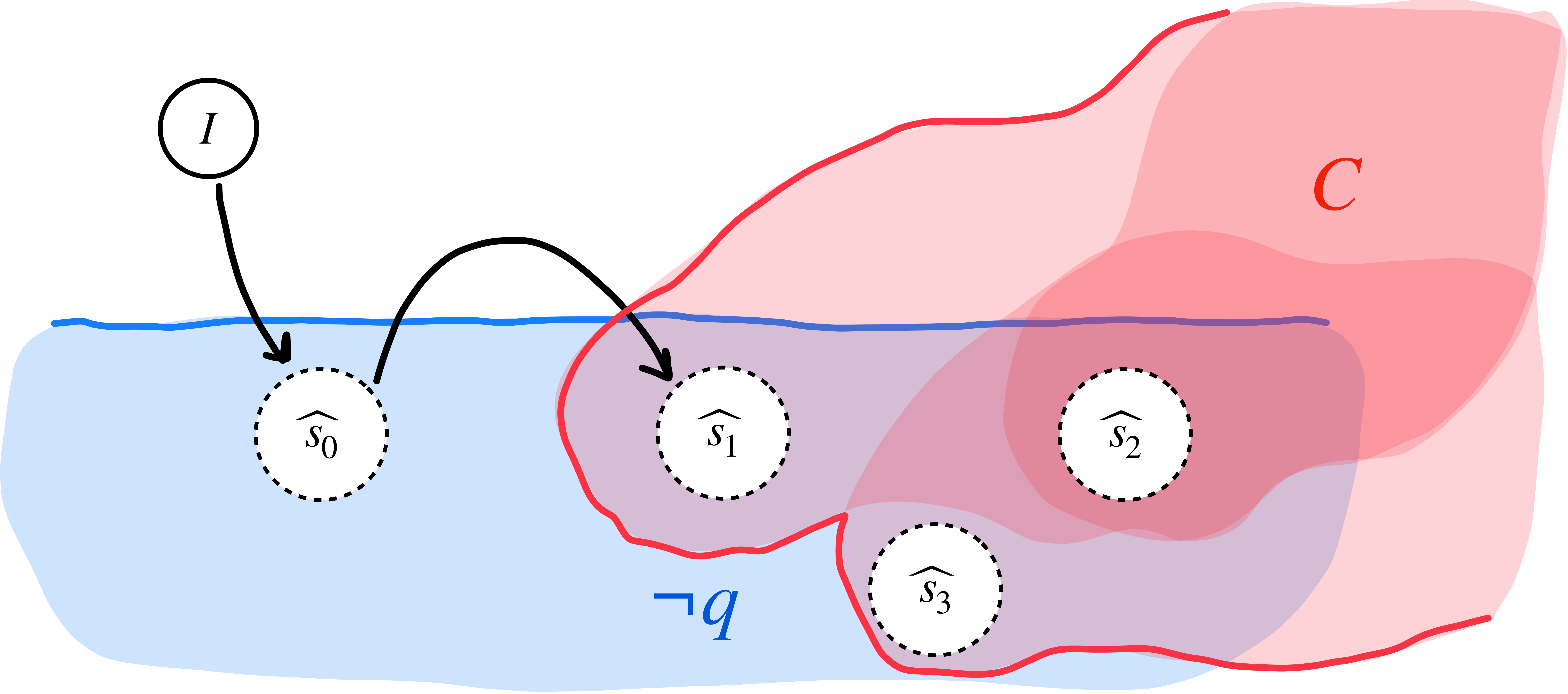

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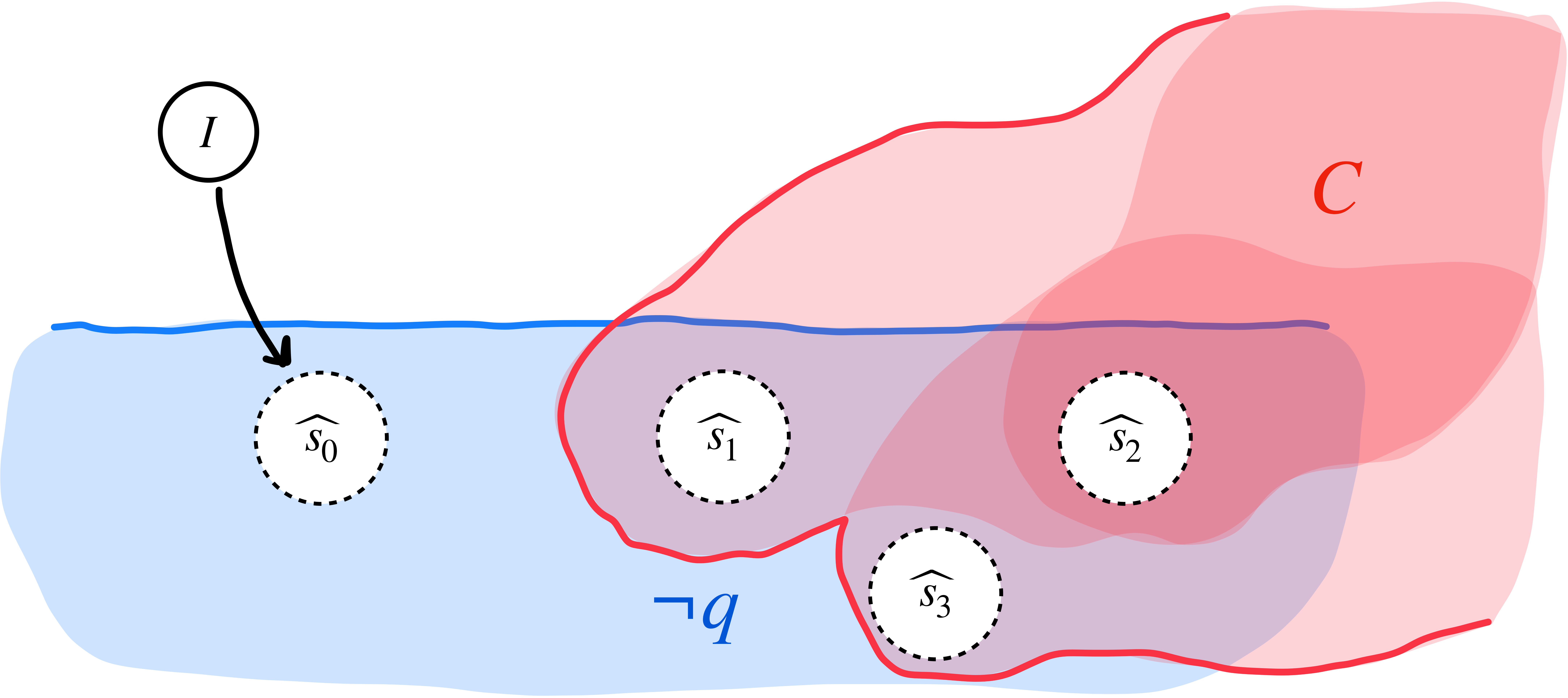

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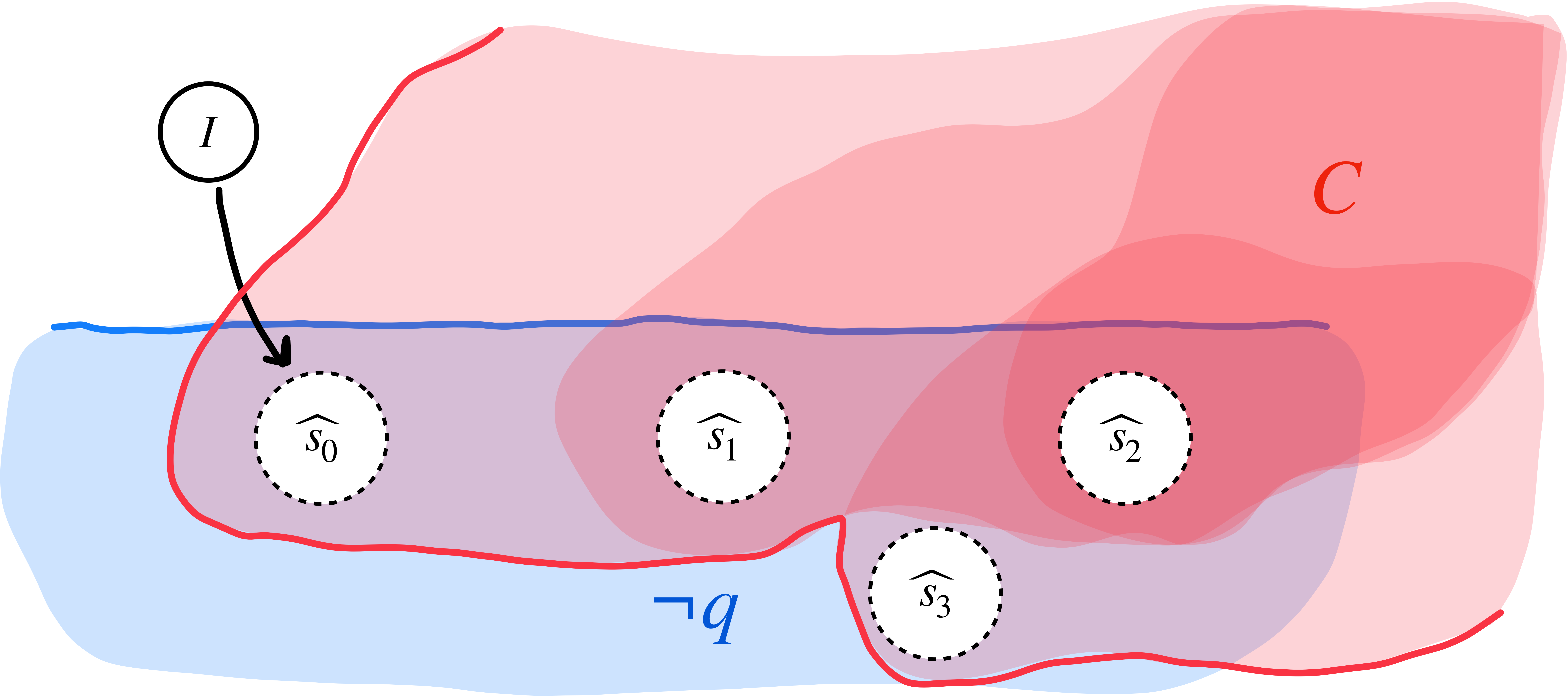

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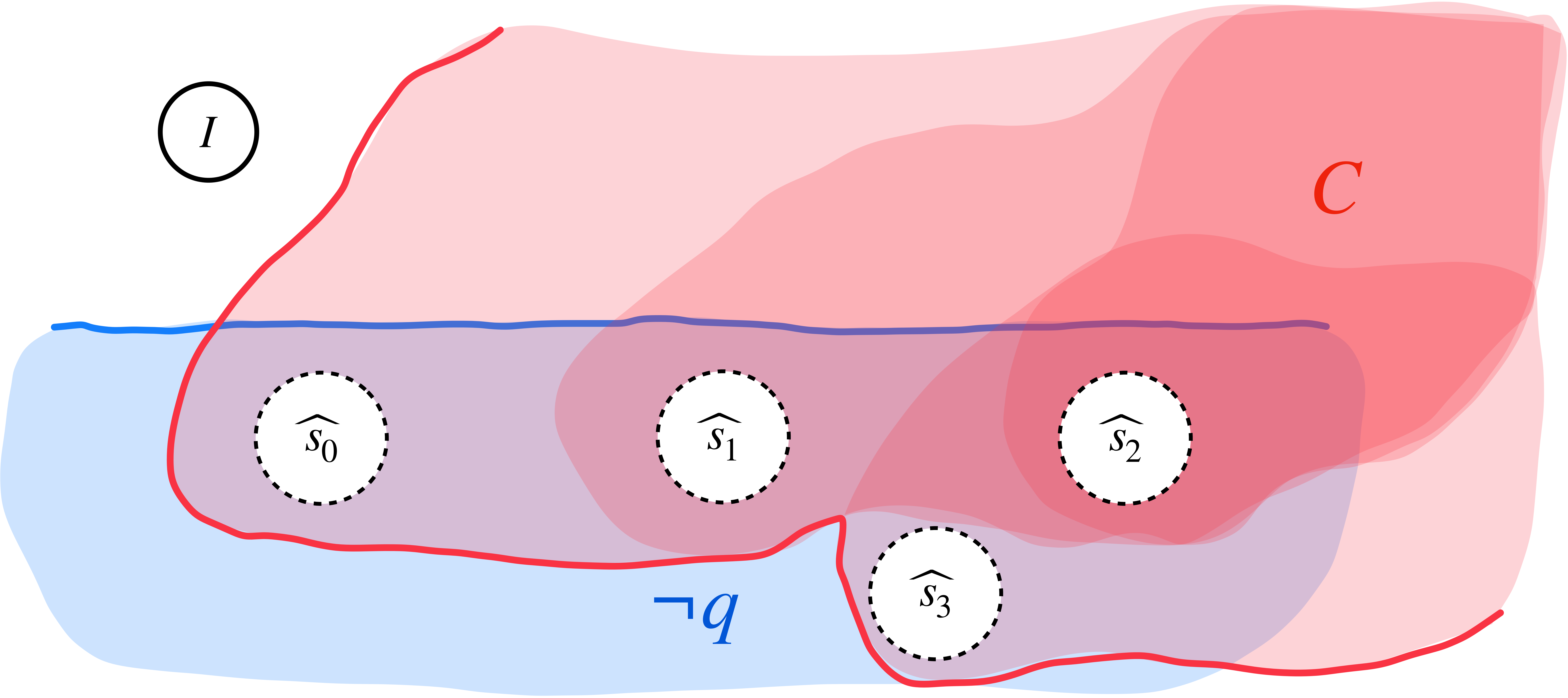

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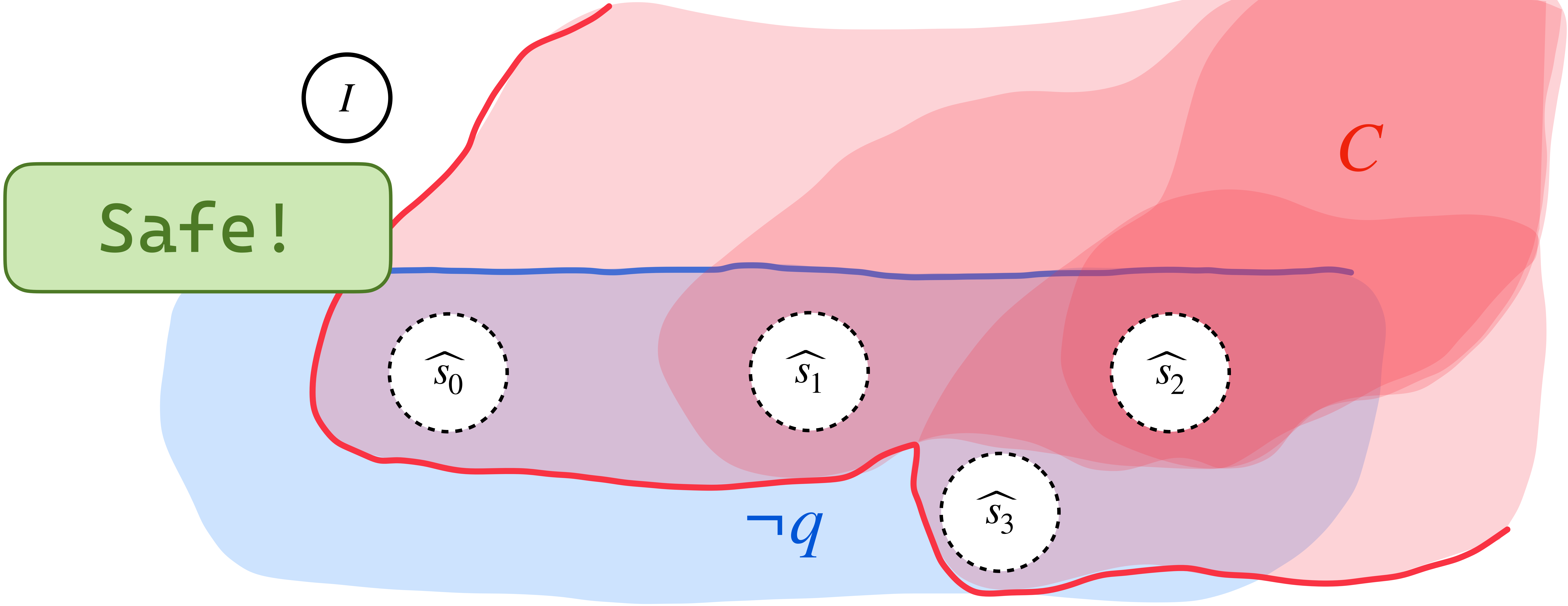

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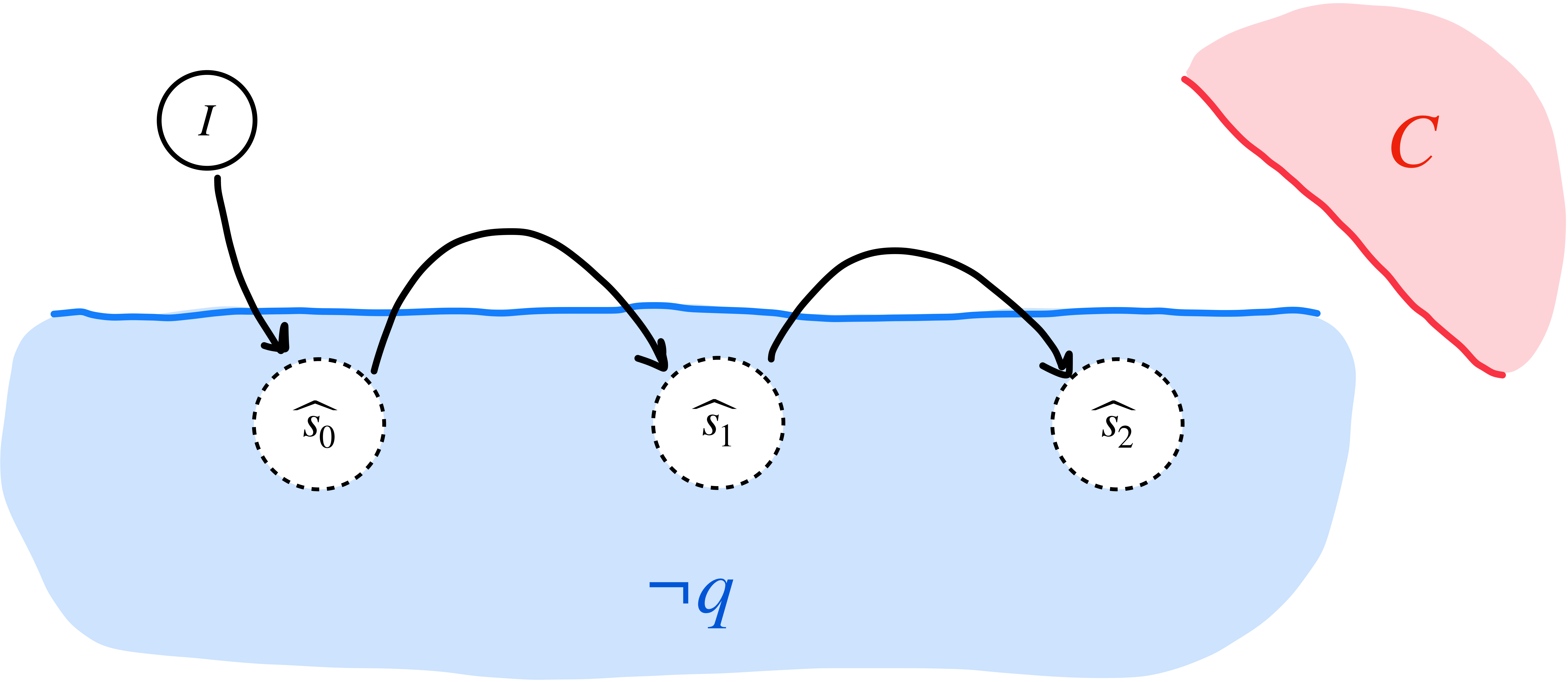

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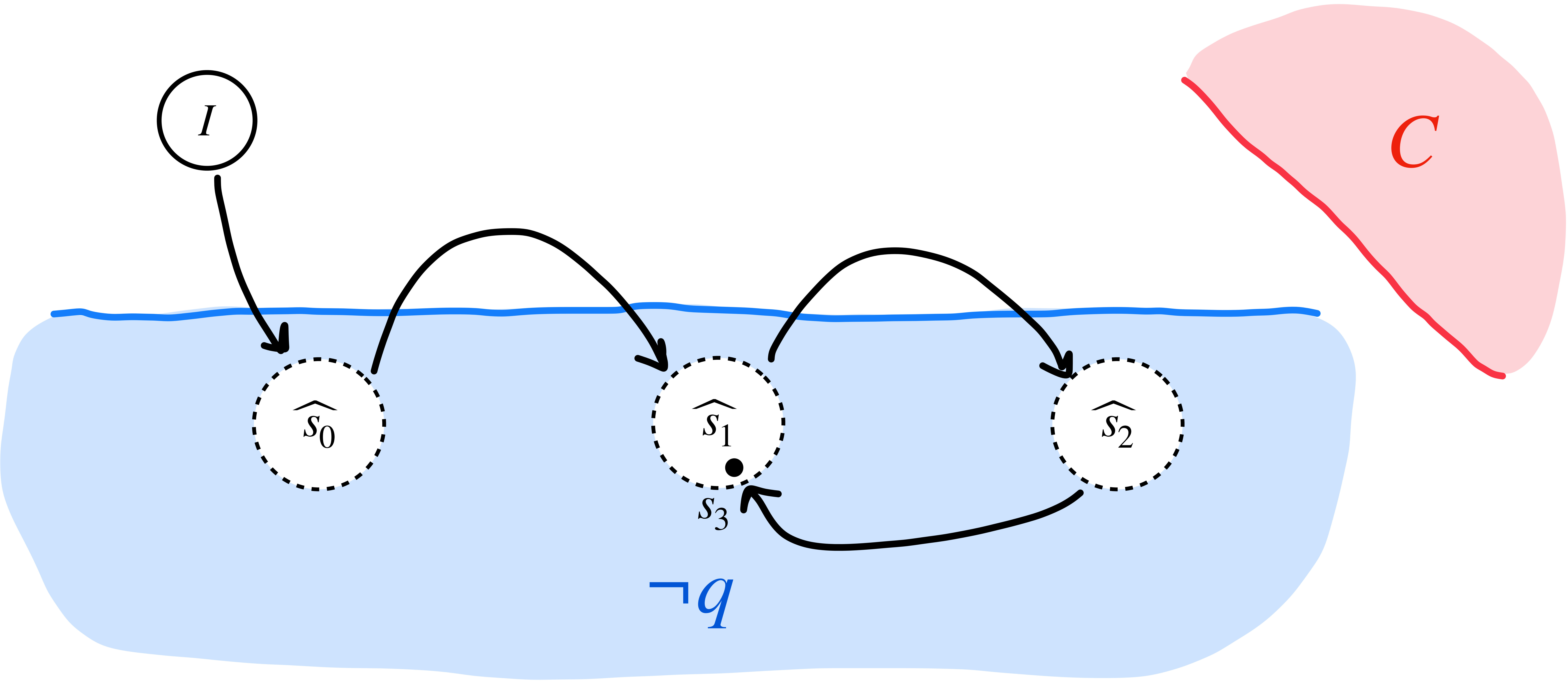

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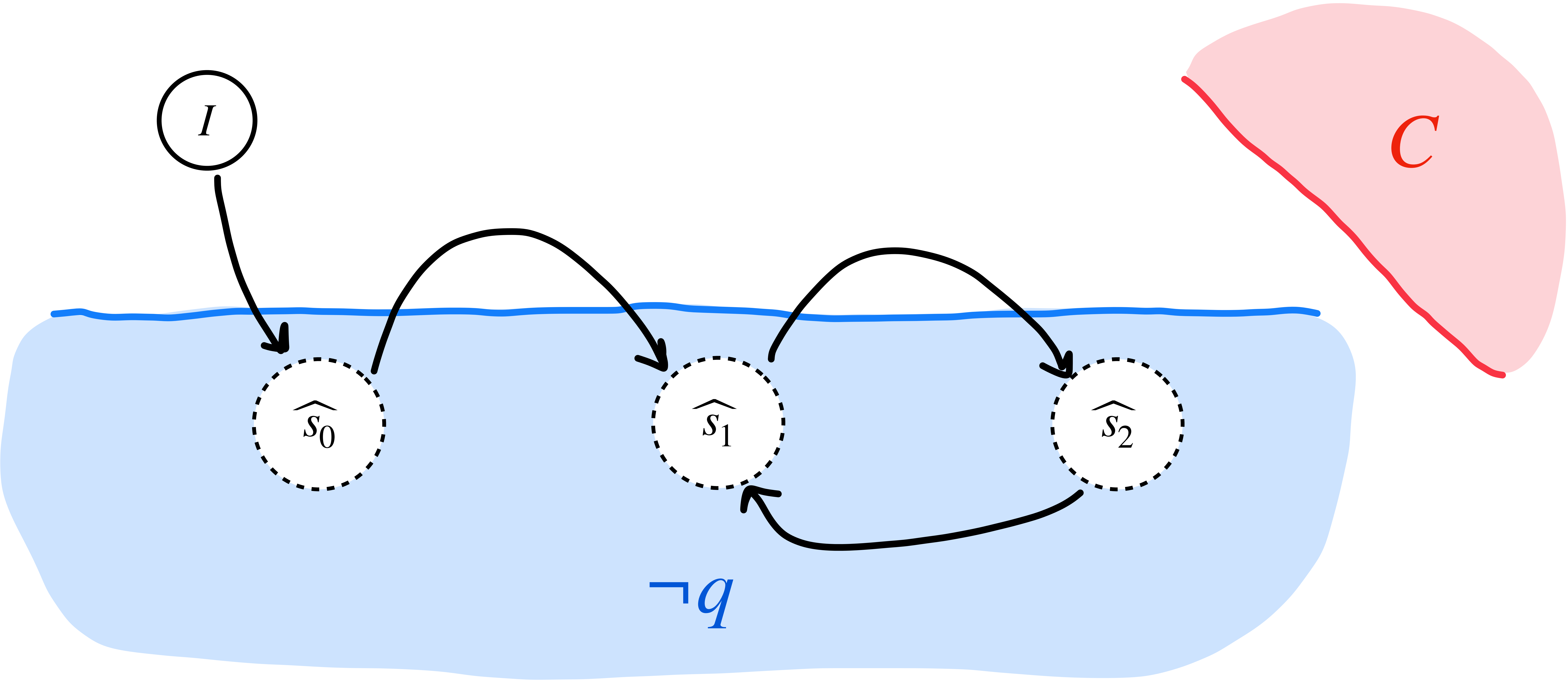


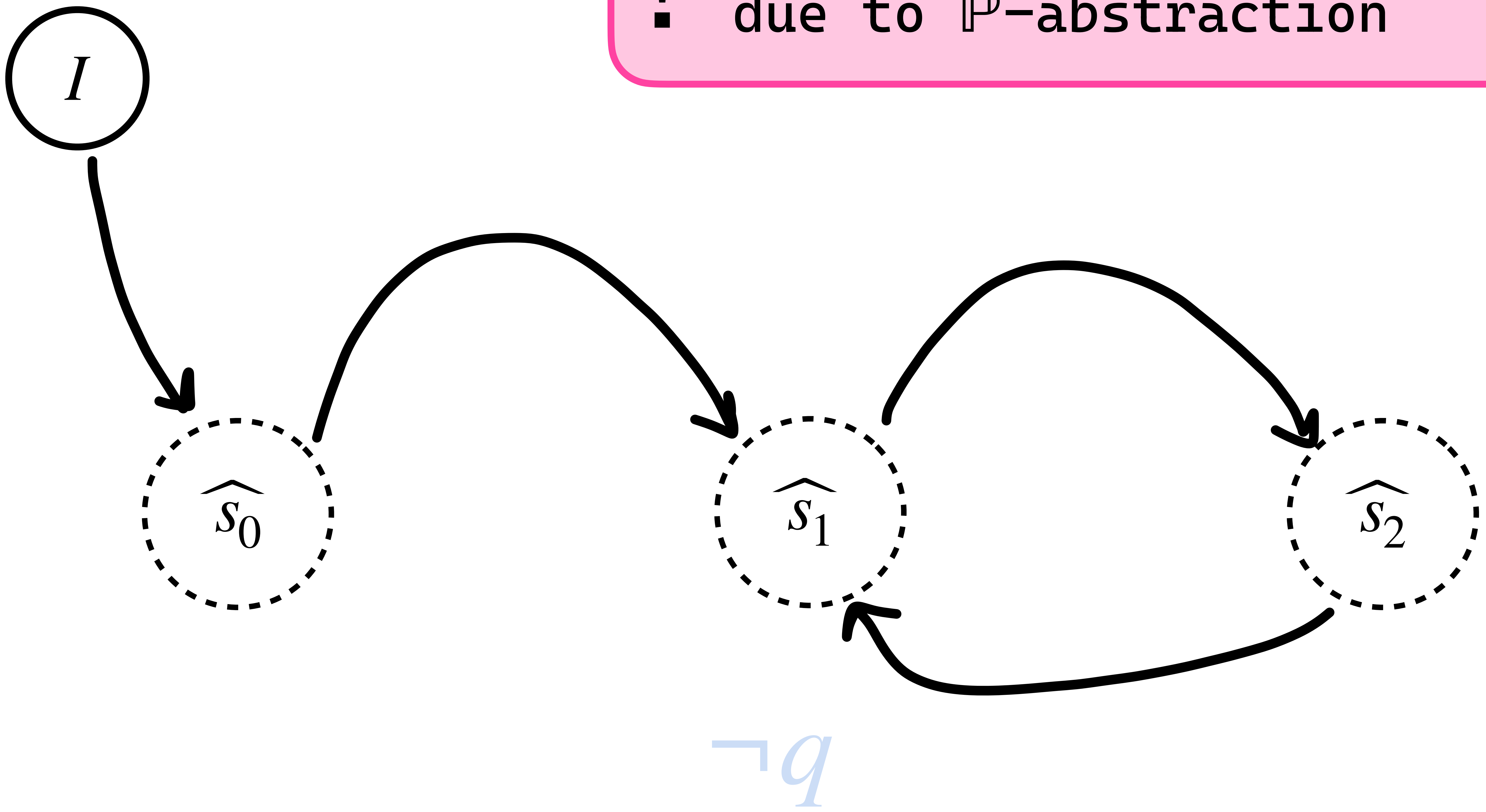
$\widehat{s_0}\ \widehat{s_1}\ \widehat{s_2}$

$\varphi = \Diamond\Box q$

$\mathbb{P} = \{p_0, p_1, \ldots\}$

! Counterexample could be spurious due to $\mathbb{P}$-abstraction

C



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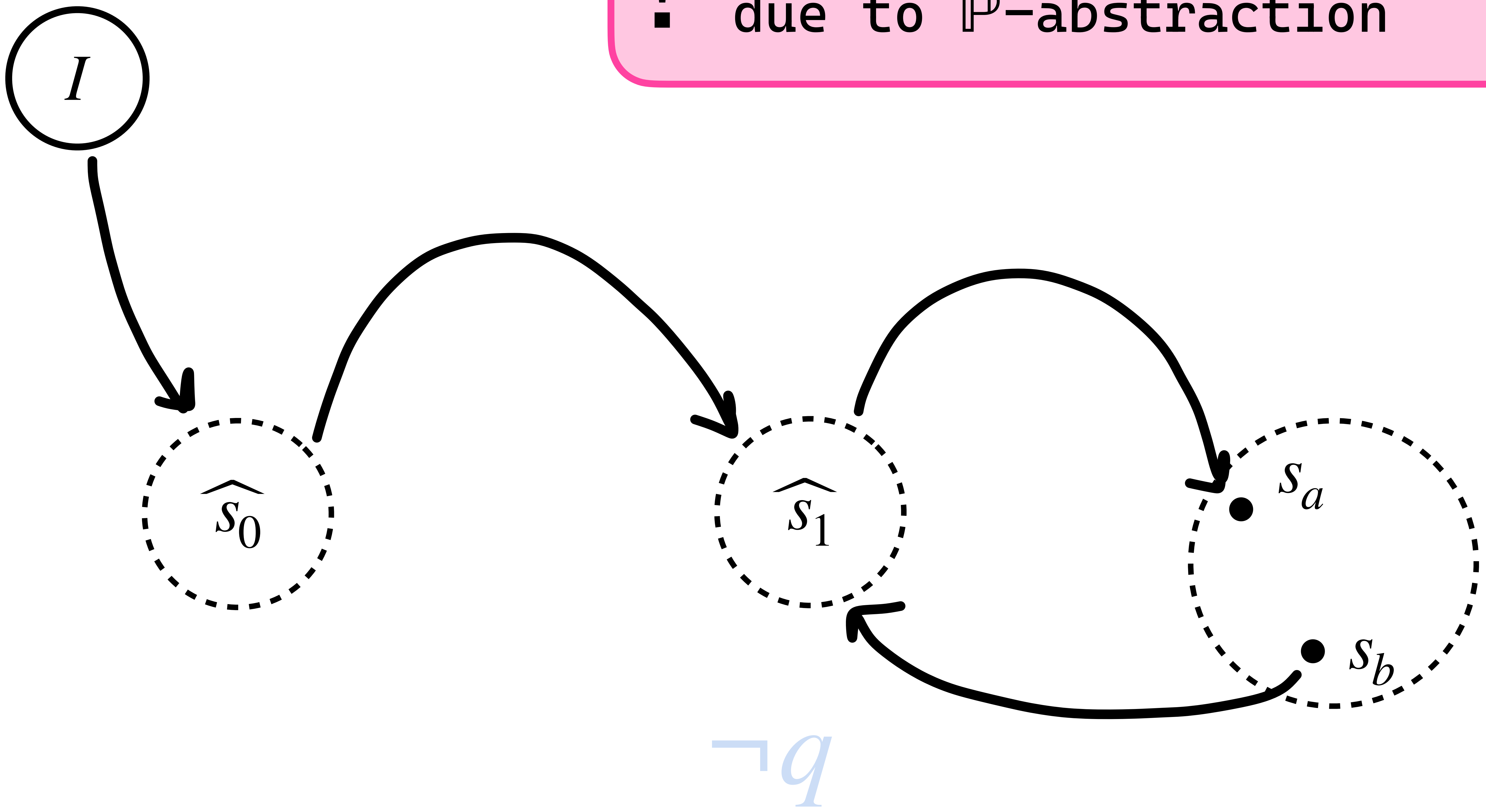

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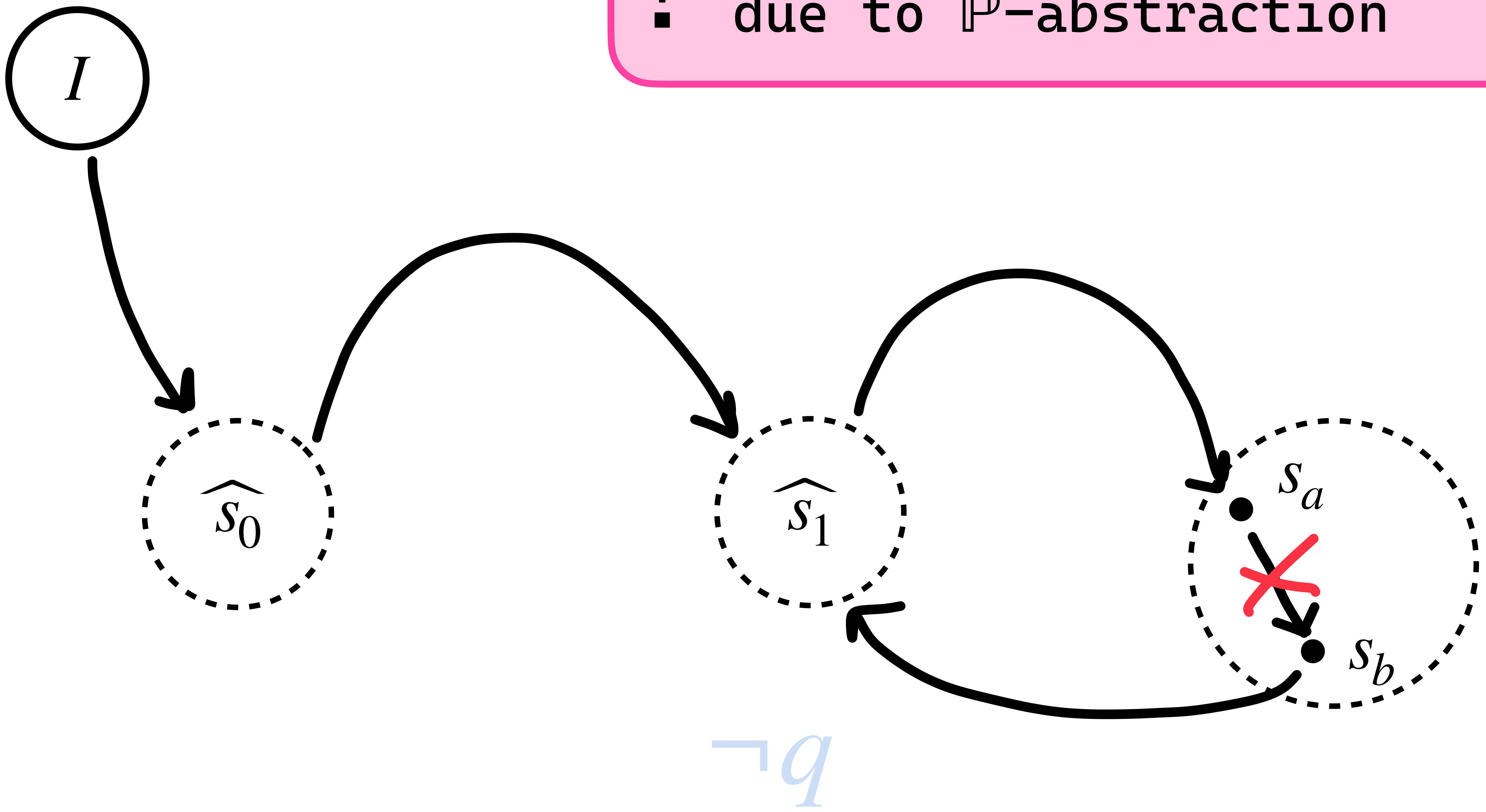

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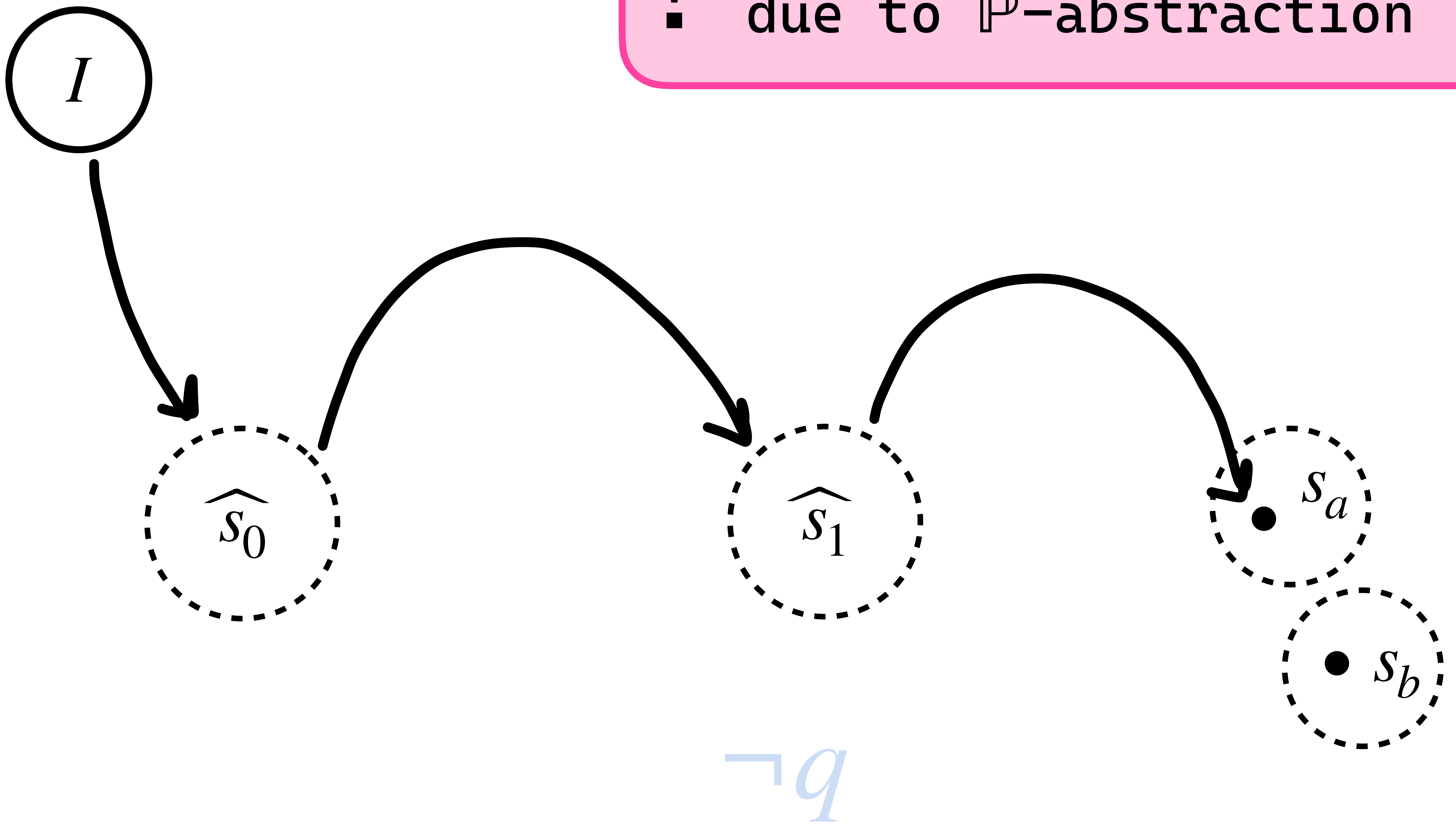

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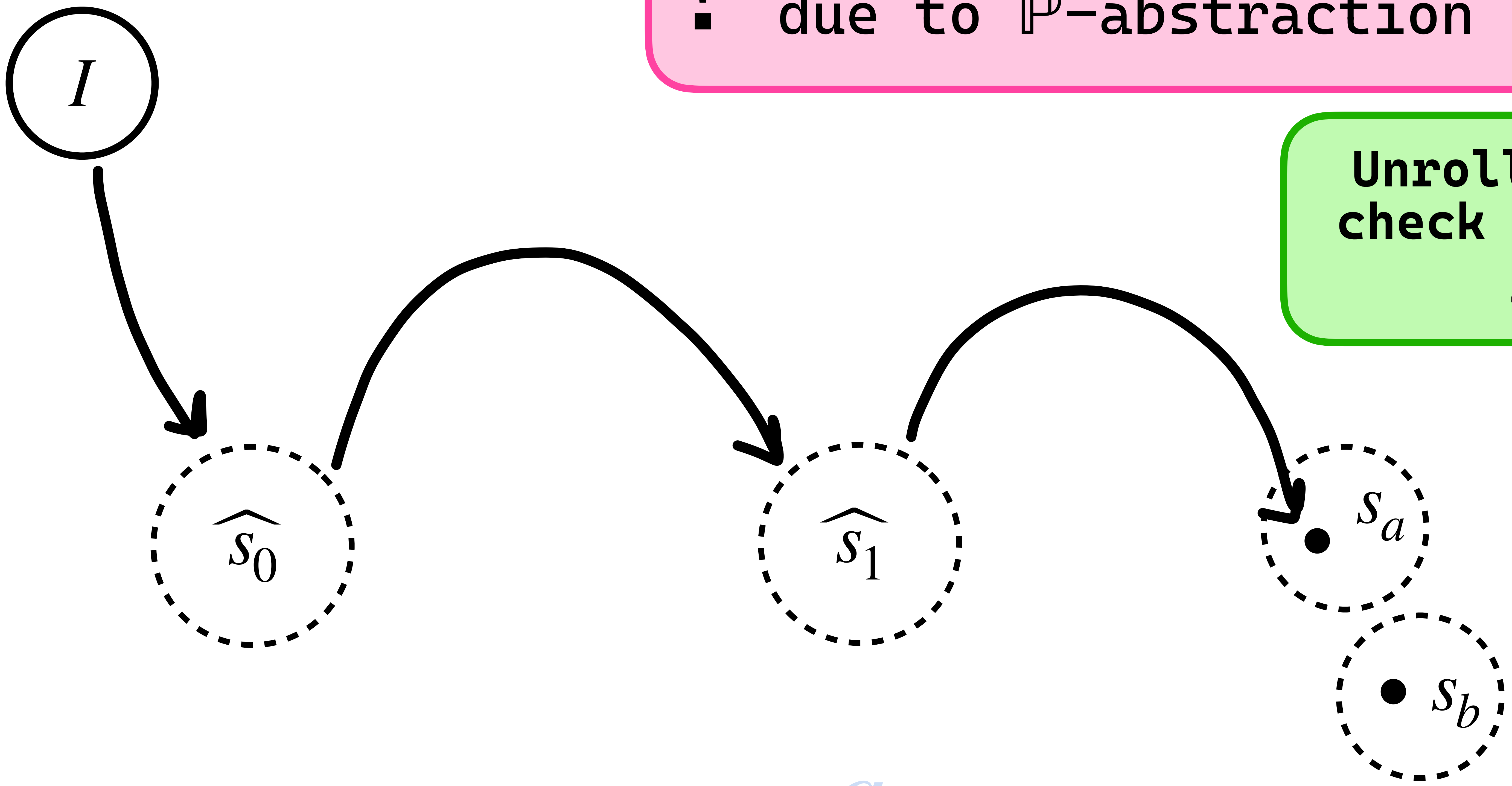
Unroll loop K times, check feasibility and refine $\mathbb{P}$.



$\neg q$

! Loop could be spurious due to having only finite but unbounded unrollings.

$\widehat{s}_0 \quad \widehat{s}_1 \quad \widehat{s}_2$

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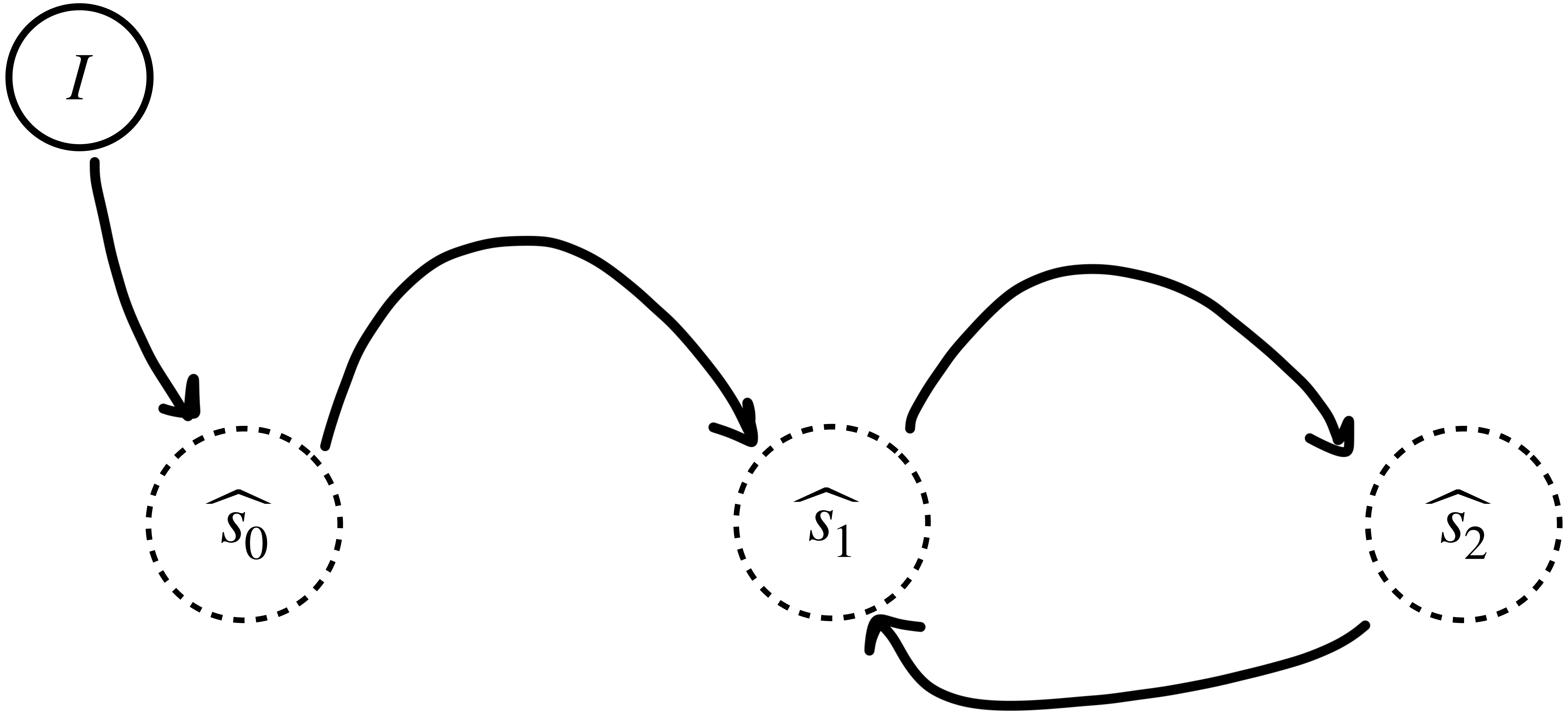

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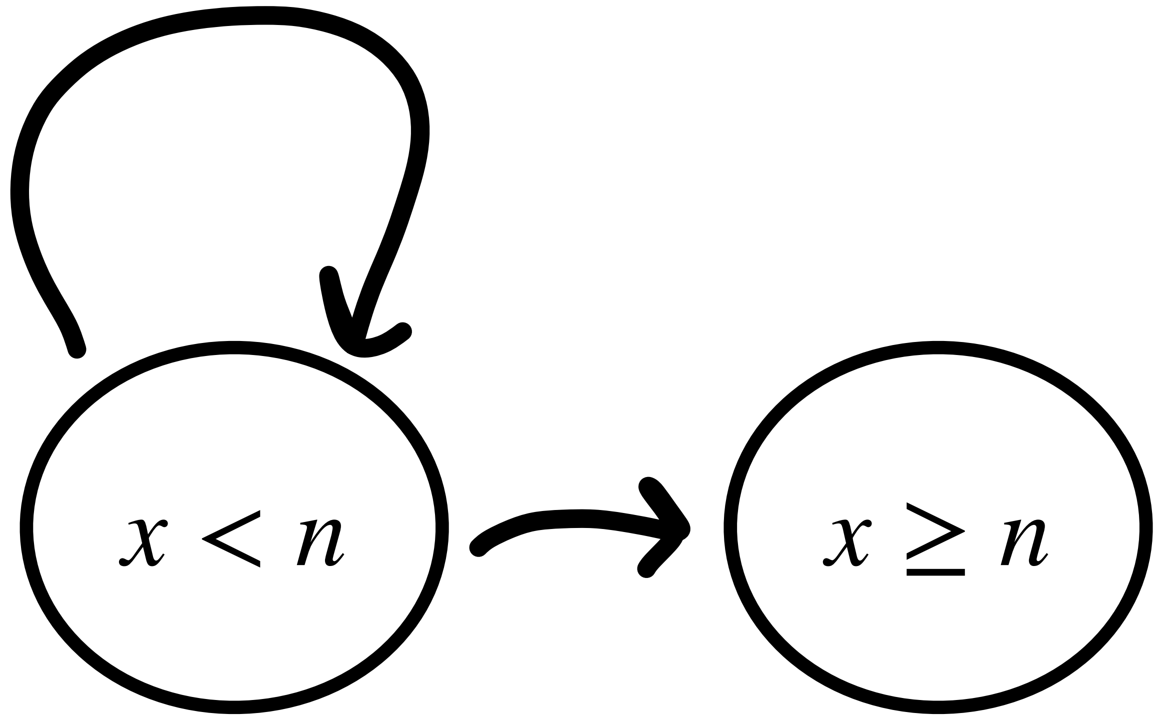

$$I := (x = 0) \land (n > x)$$
$$T := (x' = x + 1) \land (n' = n)$$
$$P := \Diamond \Box (x \geq n)$$

$\varphi = \Diamond \Box q$

$\{p_0, p_1, \ldots\}$

$\widehat{s_0}$ $\widehat{s_1}$ $\widehat{s_2}$

I

C

$\widehat{s_2}$

$\neg q$

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Well-founded Relations

$\widehat{s_0}$ $\widehat{s_1}$ $\widehat{s_2}$

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Well-founded Relations

Given a set of states Q, a **well-founded relation (WFR)** $w : Q \times Q$ is a binary relation such that every non-empty subset $U \subseteq Q$ has a minimal element $m \in U$ (there is no $u \in U$ such that $(m, u) \in w$).

C

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If there exists a WFR w over the states in π_{loop}, then π_{loop} cannot be unrolled infinitely (w is a proof of π_{loop}'s spuriousness).

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Use off-the-shelf methods

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$\widehat{s_0}$ $\widehat{s_1}$ $\widehat{s_2}$

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If there exists a WFR w over the states in π_{loop}, then π_{loop} cannot be unrolled infinitely (w is a proof of π_{loop}'s spuriousness).

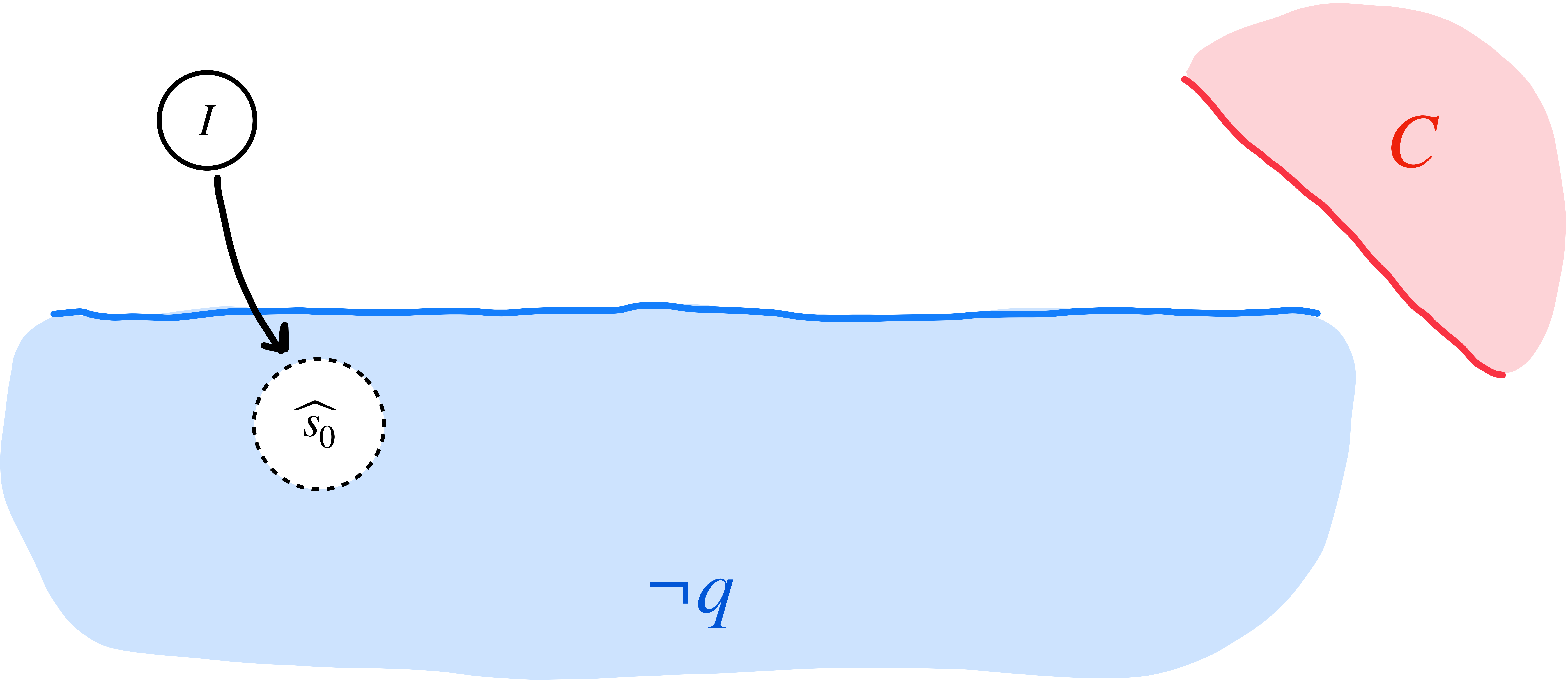
Given a set of WFRs $\mathbb{W}$, if π_{loop} is part of a genuine counterexample, then there is some pair of states $(s_0, s_1) \in \pi_{loop}$ such that (s_0, s_1) are not in any WFR $w \in \mathbb{W}$.

C

Use off-the-shelf methods

$\widehat{s_0}$

$\varphi = \Diamond \Box q \quad \mathbb{P} = \{p_0, p_1, \ldots, p_n\} \quad \mathbb{W} = \{w_0, w_1, \ldots w_m\}$



! How to block spurious loops using $\mathbb{W}$?

$\widehat{s_0}$

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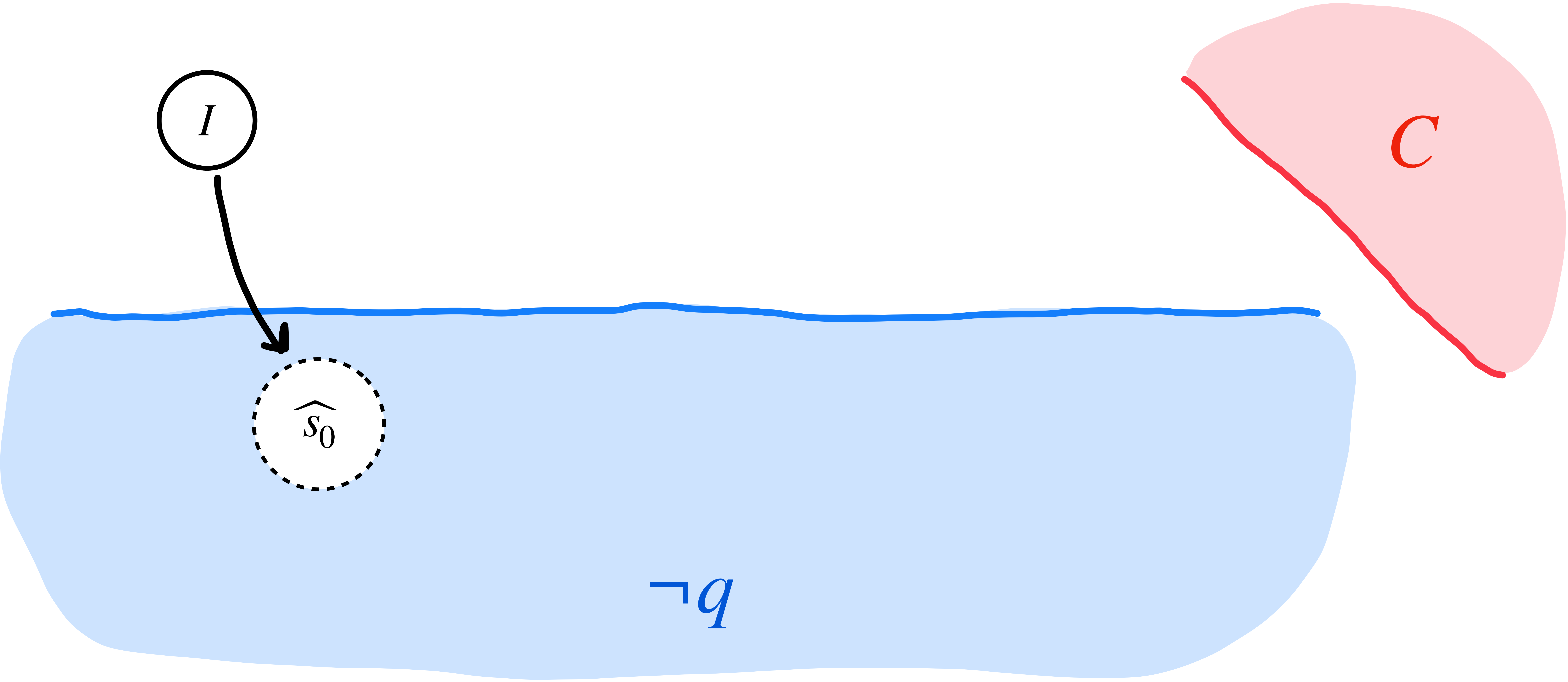

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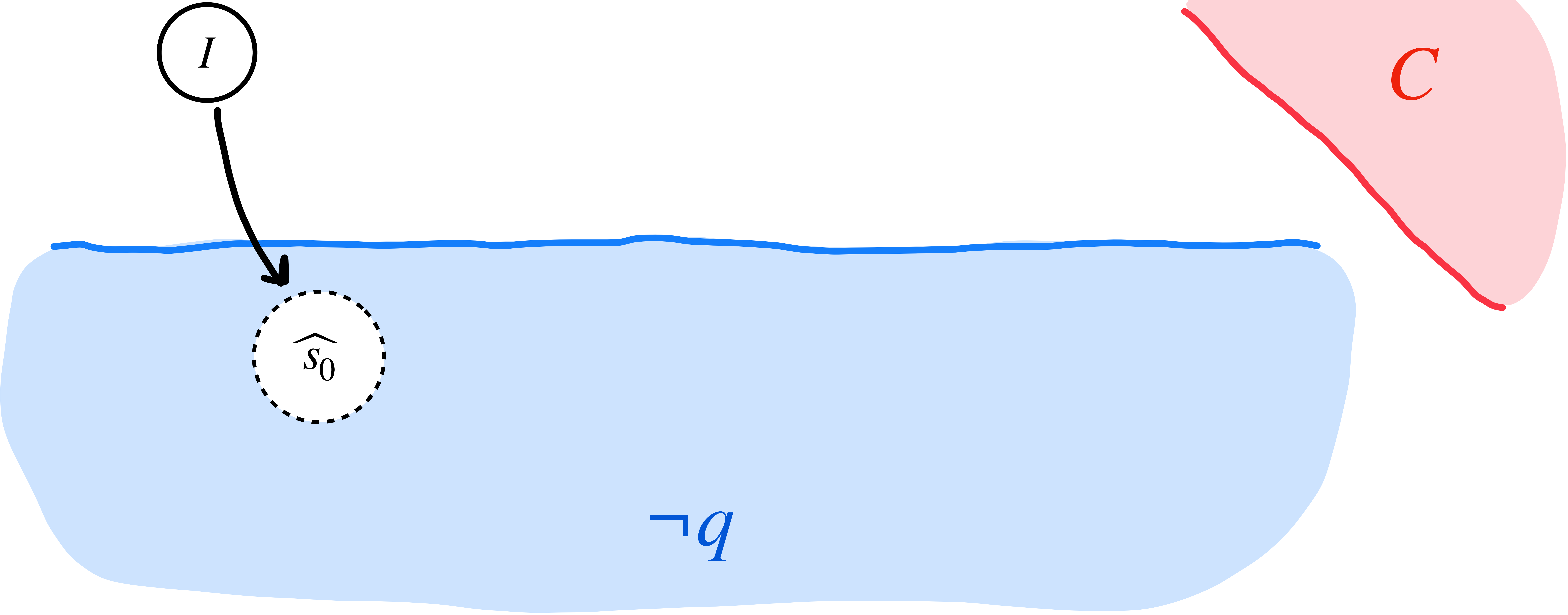
Search for **pair** of $\neg q$-states not in any $w \in \mathbb{W}$.

$$\mathbb{W} = \{w_0, w_1, \dots w_m\}$$



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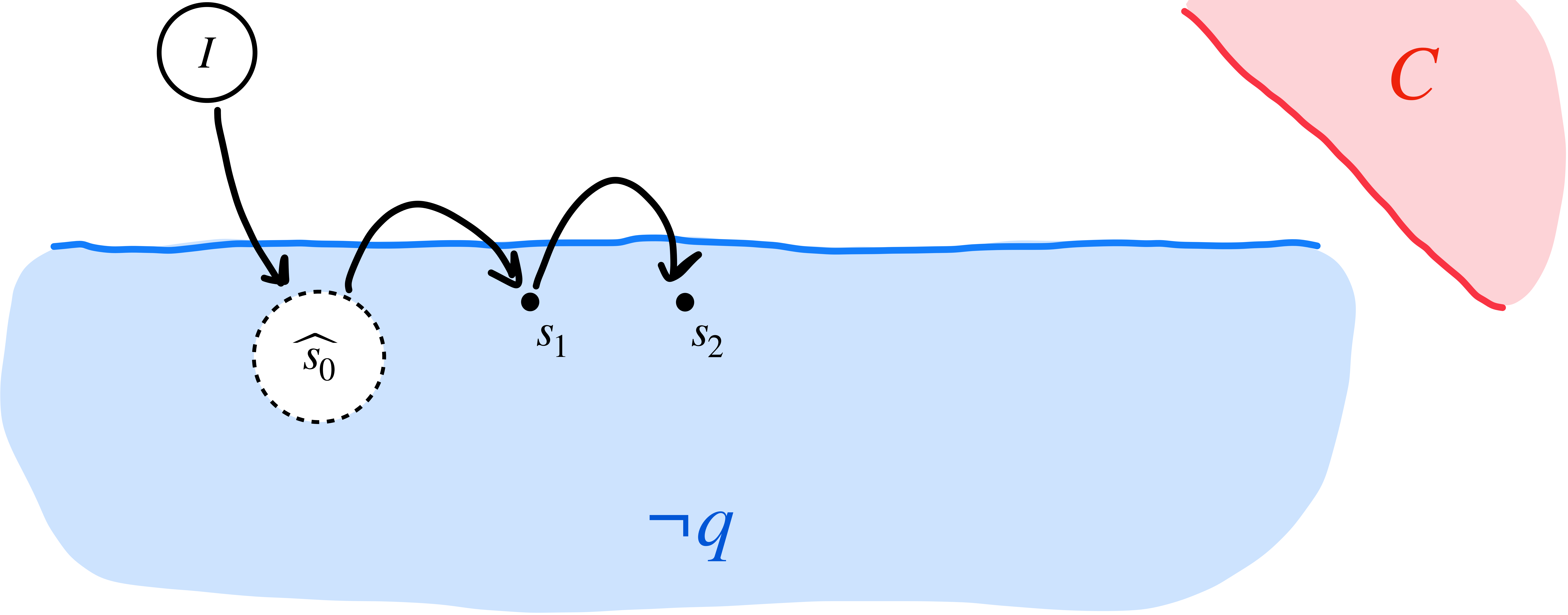

! How to block spurious loops using $\mathbb{W}$?

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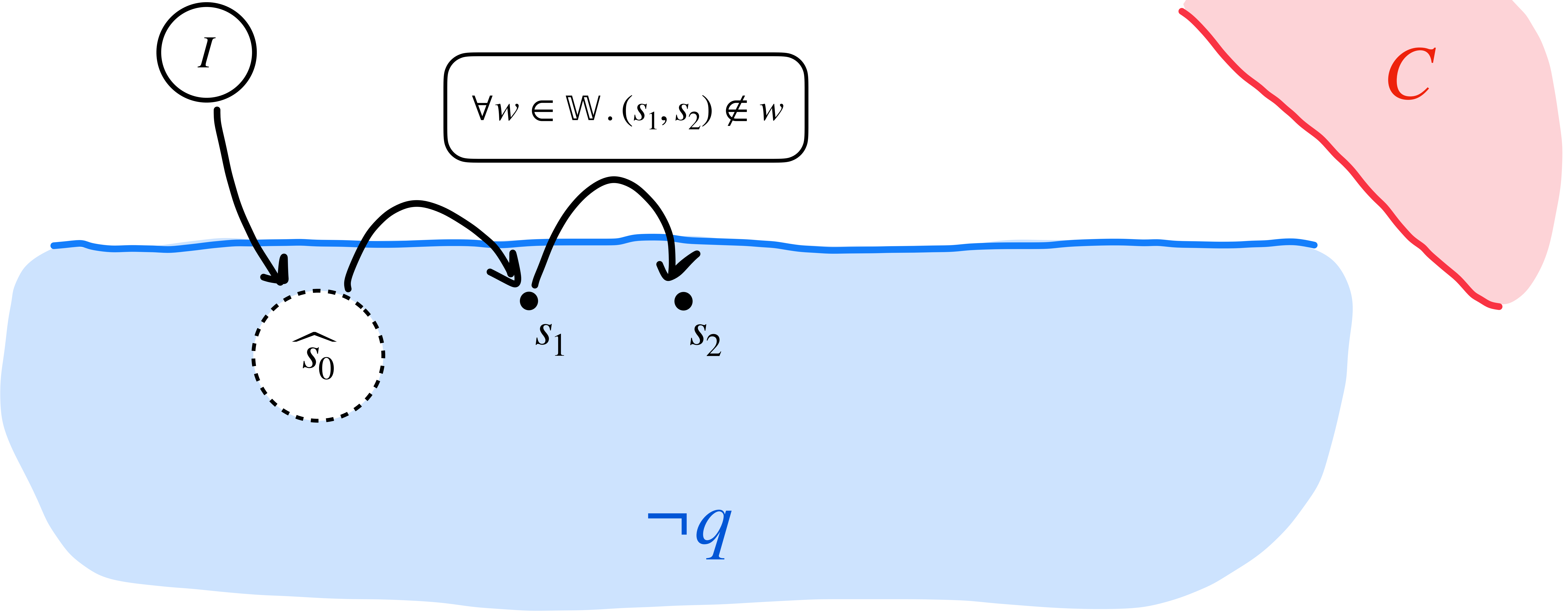

$S = \langle X, I(X), T(X, X') \rangle$ **Search Stack:** $\widehat{s_0}$ $\widehat{s_2}$

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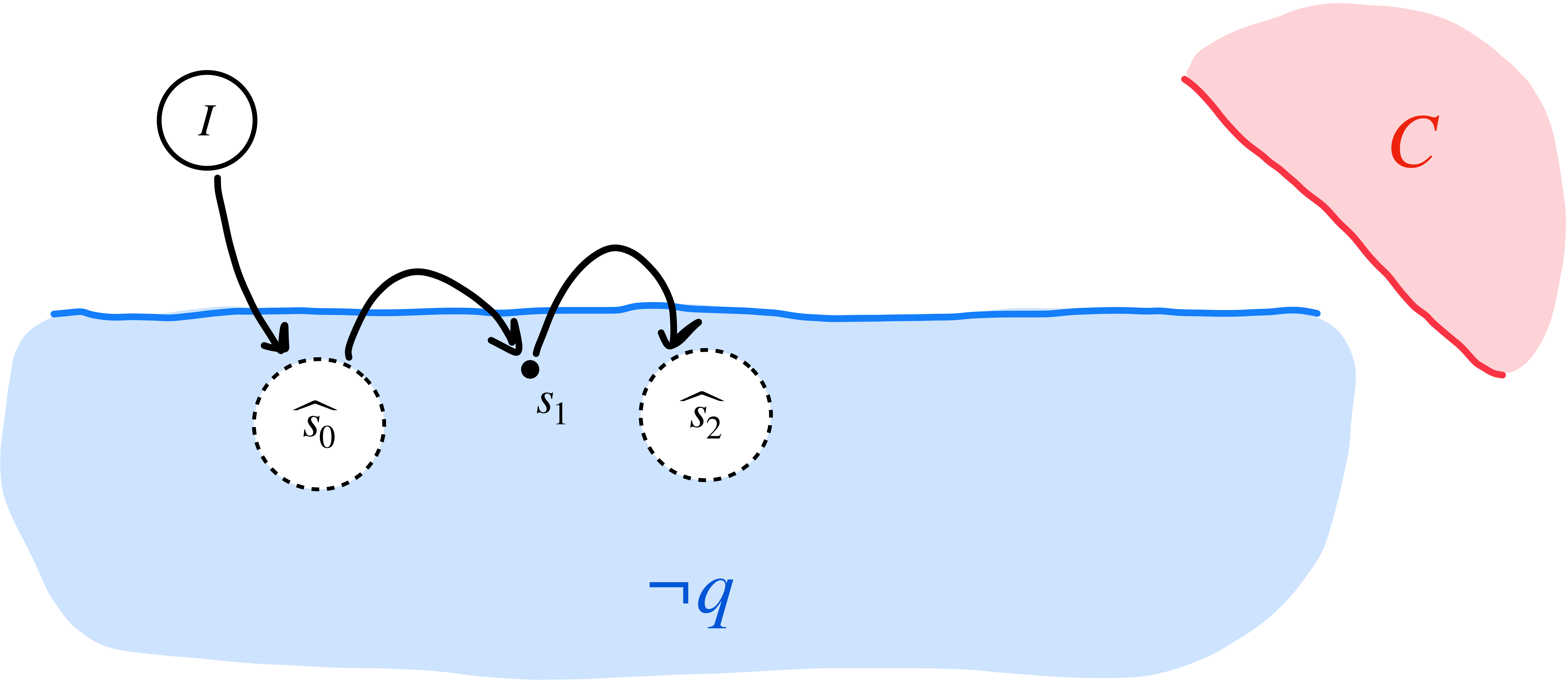

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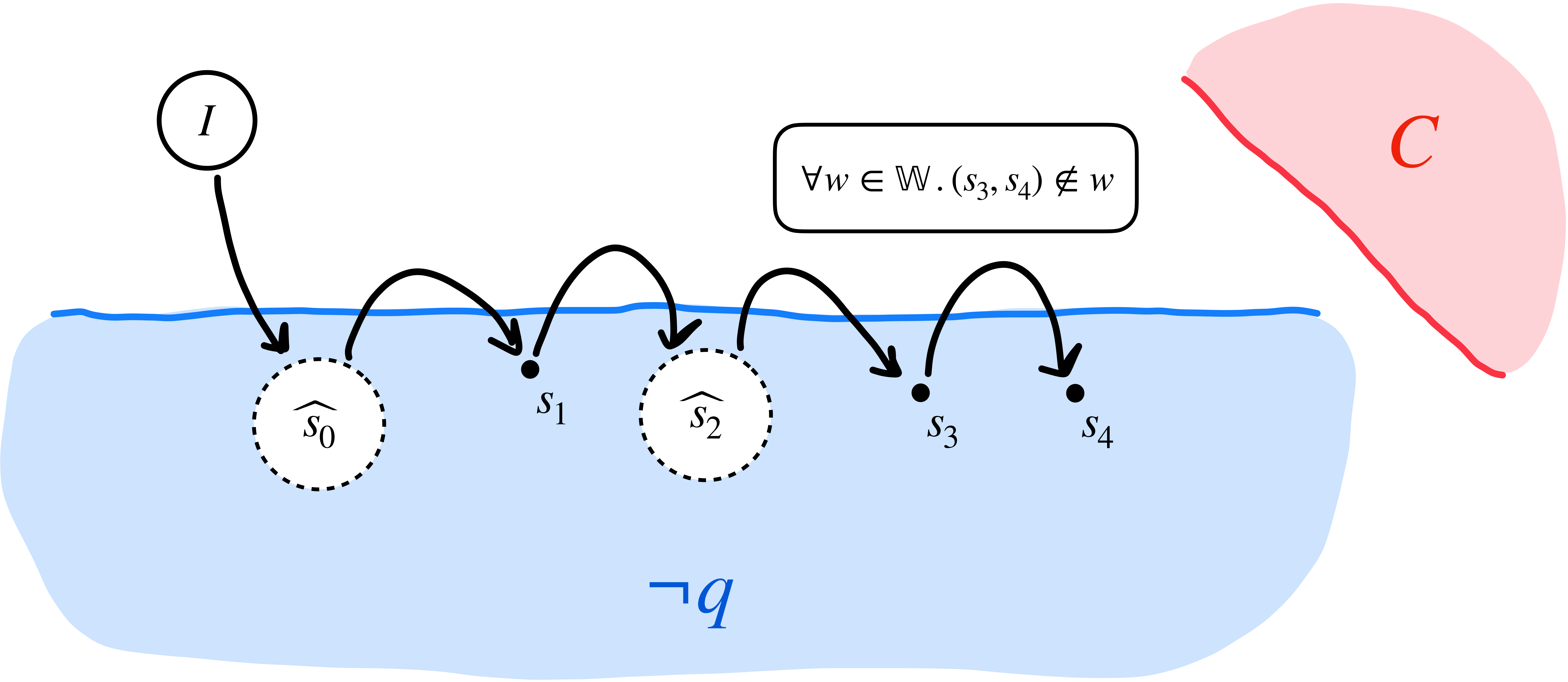

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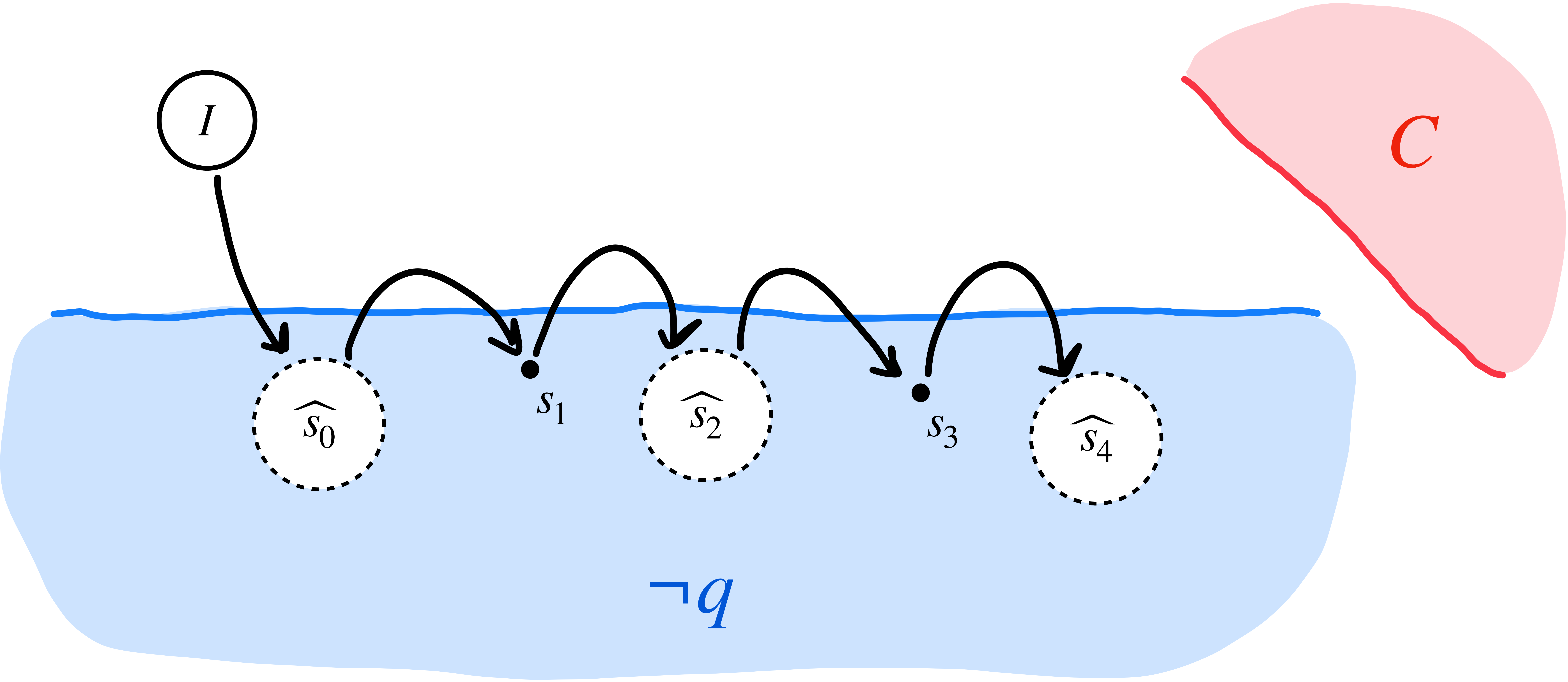

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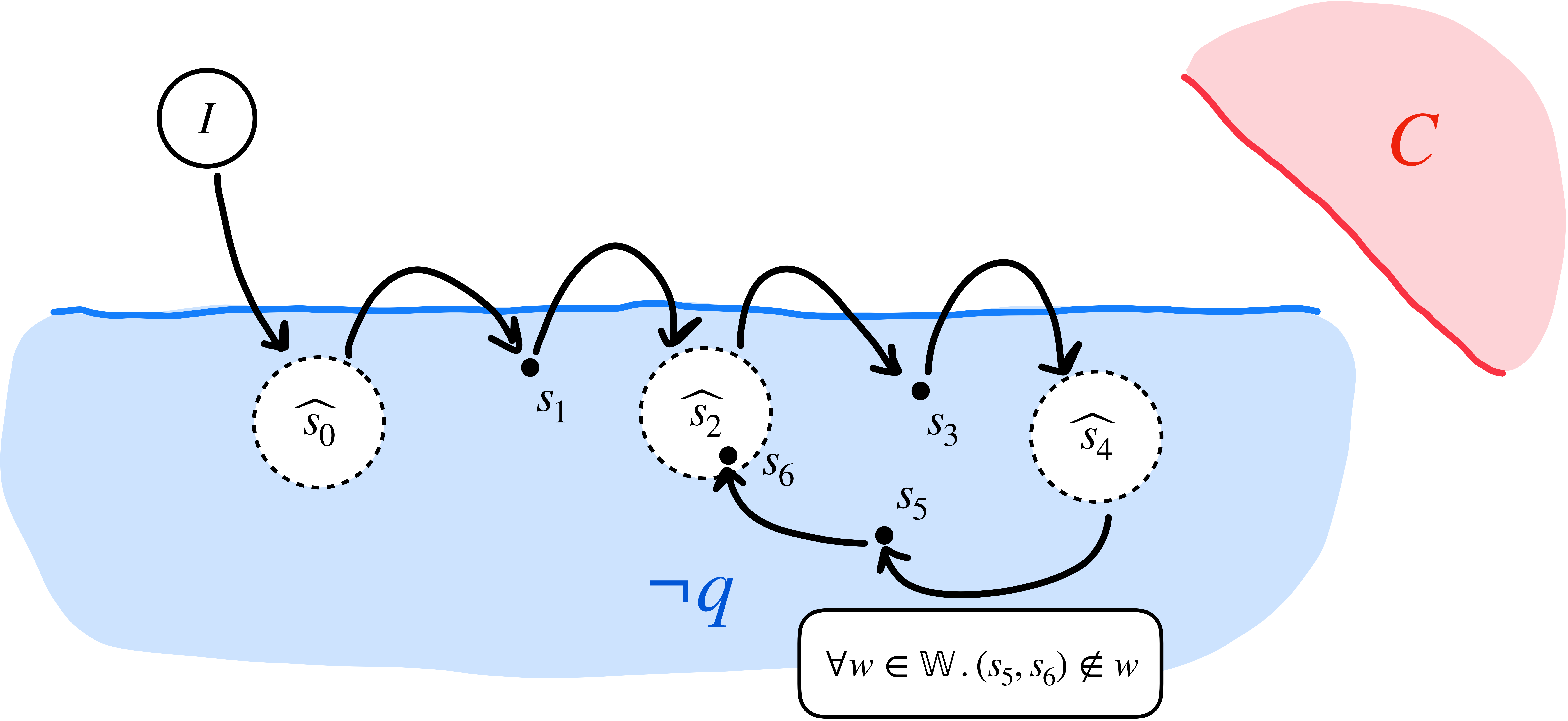

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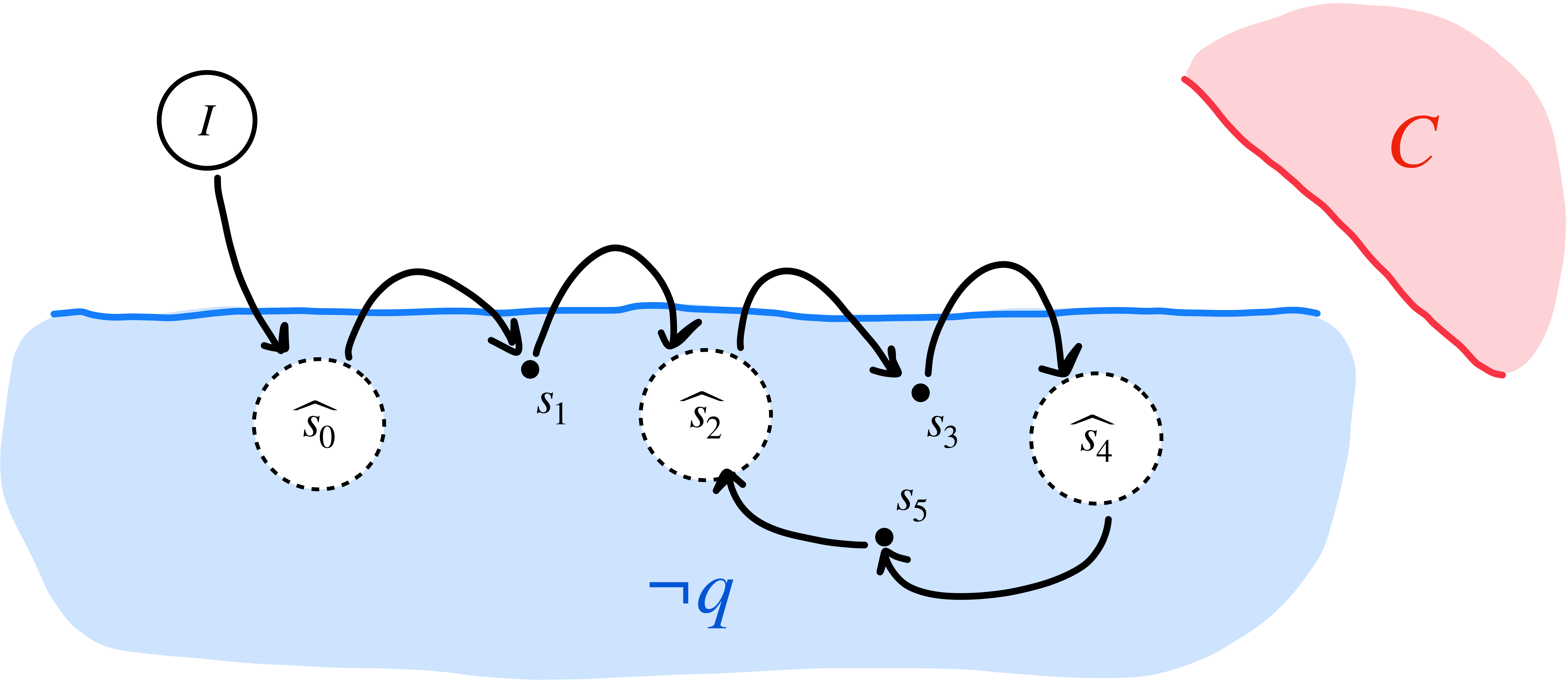

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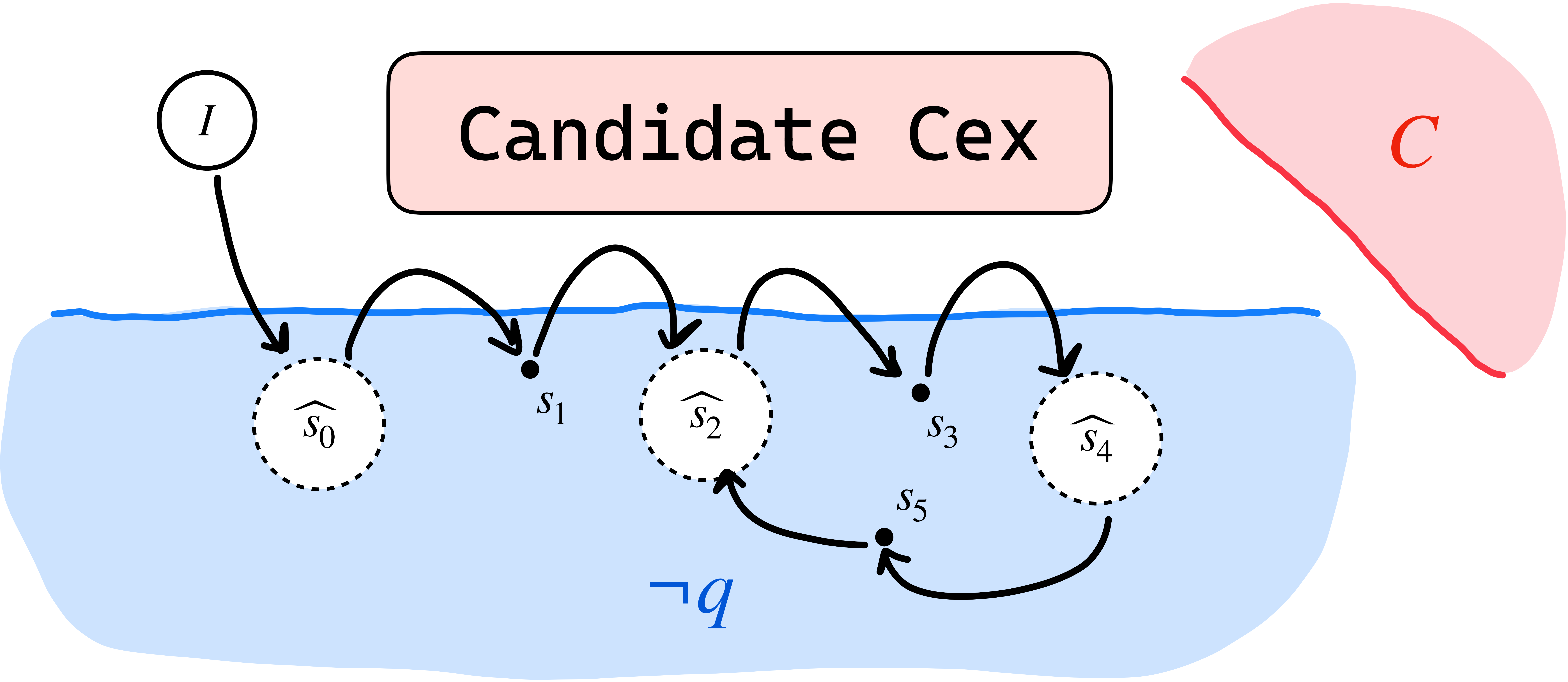

RLive-inf Overview

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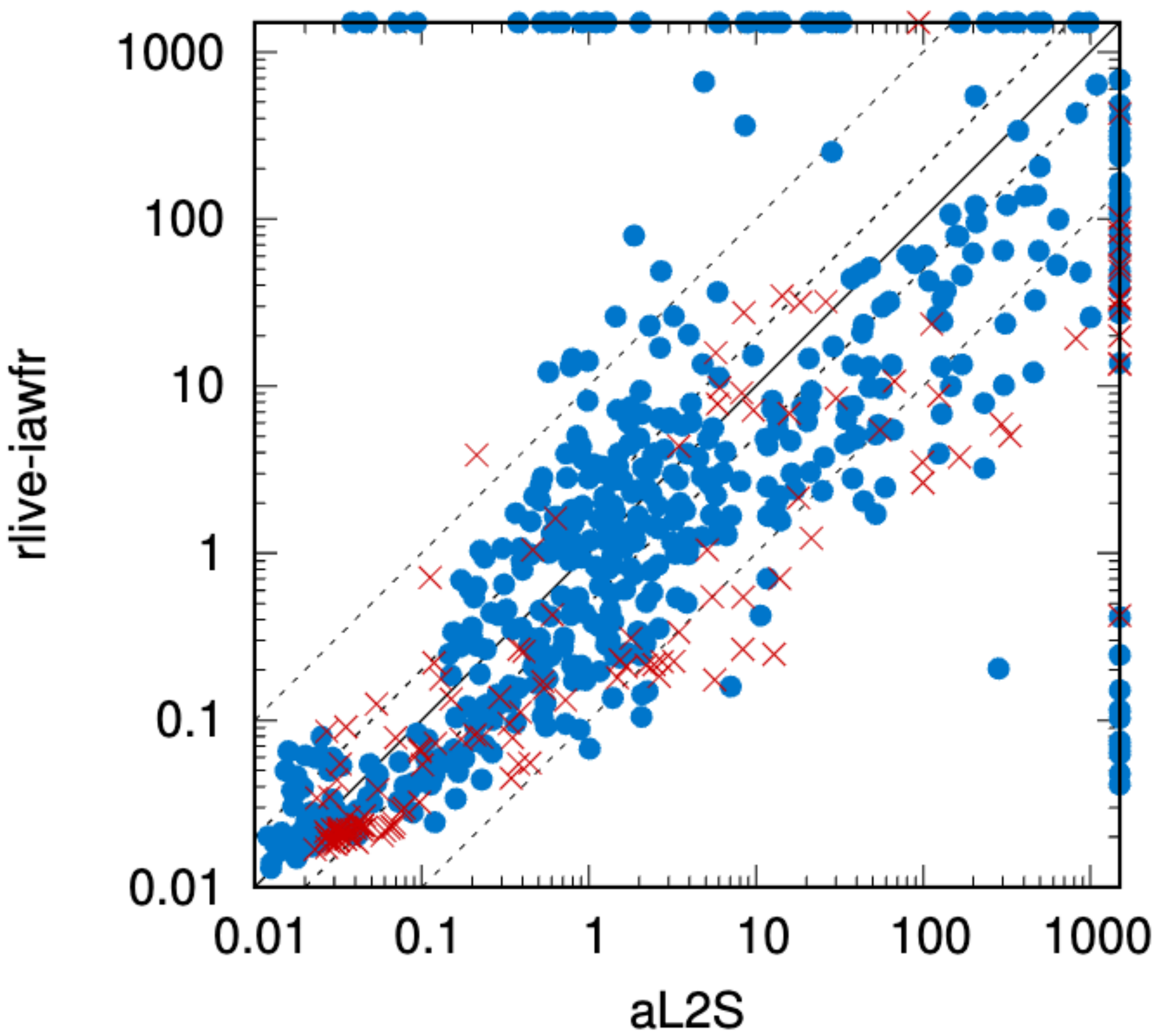
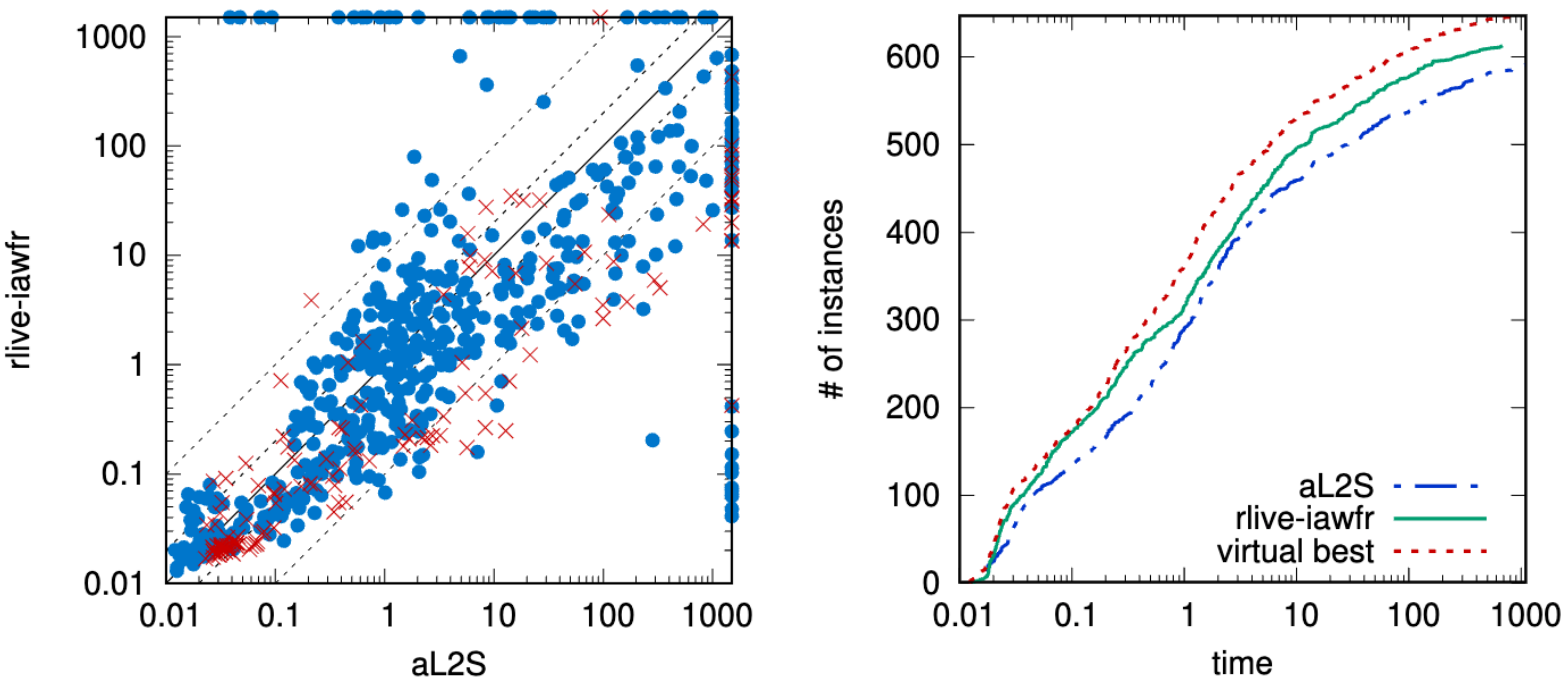
- Benchmarks:
  - Symbolic transition systems (556)
  - Imperative-style programs (86)
  - Termination benchmarks (193)



rlive-iawfr
aL2S
1000
100
10
1
0.1
0.01
0.01
0.1
1
10
100
1000



1000
100
10
1
0.1
0.01
rlive-iawfr
0.01
0.1
1
10
100
1000
aL2S
600
500
400
300
200
100
0
# of instances
0.01
0.1
1
10
100
1000
time
aL2S
rlive-iawfr
virtual best

RLive-inf Summary

- New algorithm for **infinite-state liveness checking**.
- Finds candidate counterexamples via **incremental, depth-first** search with predicate abstraction.
- Uses **well-founded relations** to block finite but unbounded loops in target system.
- Implementation **outperforms aL2S** on variety of benchmarks.

Artifact

	Num Solved (/835)	Num Safe Solved	Num Unsafe Solved	Time (s) (Non-timeouts)			Num Predicates			Num WFRs		
				Min	Max	Avg	Min	Max	Avg	Min	Max	Avg
Abstract L2S	591	470	121	0.008	1088.96	37.328	4	255	46.091	0	14	0.680
RLive-inf	613	480	133	0.006	703.248	21.983	1	94	24.278	0	15	0.675
Virtual Best	647	513	134	0.008	972.825	23.418	1	173	33.023	0	13	0.625