



# Dynamic Set Reasoning

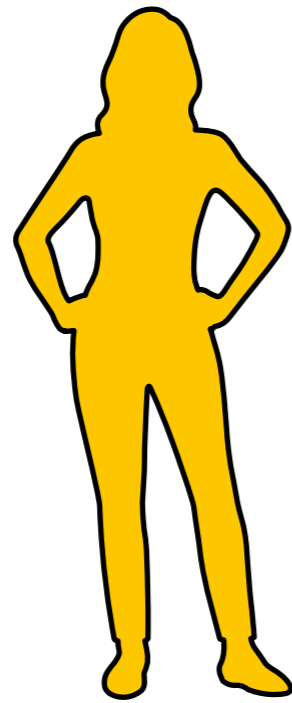
SPECIFYING AND OPTIMIZING MONITOR ENCODINGS

*Chris Johannsen*

IOWA STATE UNIVERSITY

# Sam the Satellite Engineer

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**Sam**

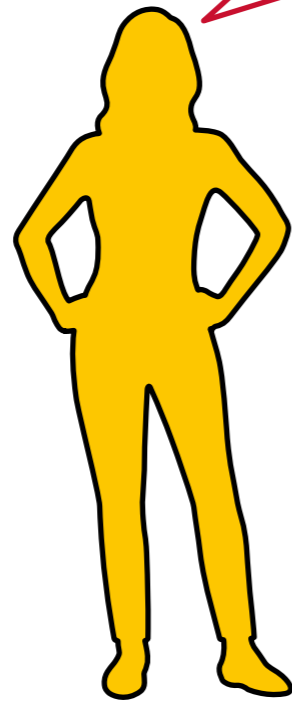


**[2]**

# Sam the Satellite Engineer

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**Stuff happens**



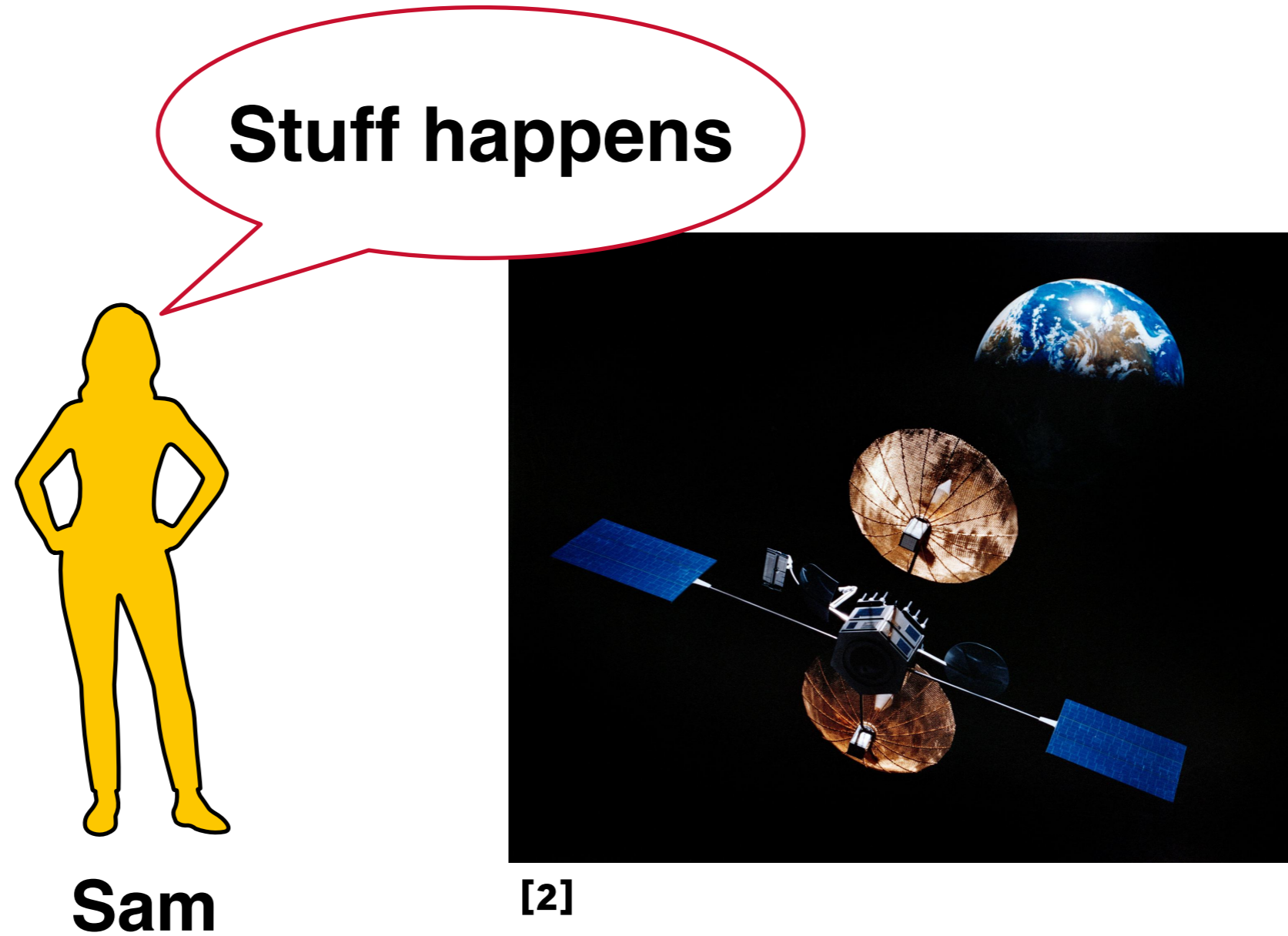
**Sam**



**[2]**

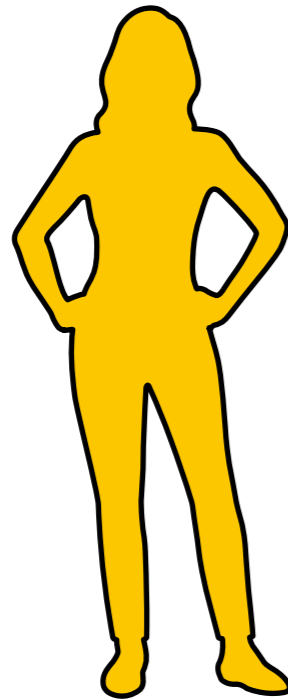
# Sam the Satellite Engineer

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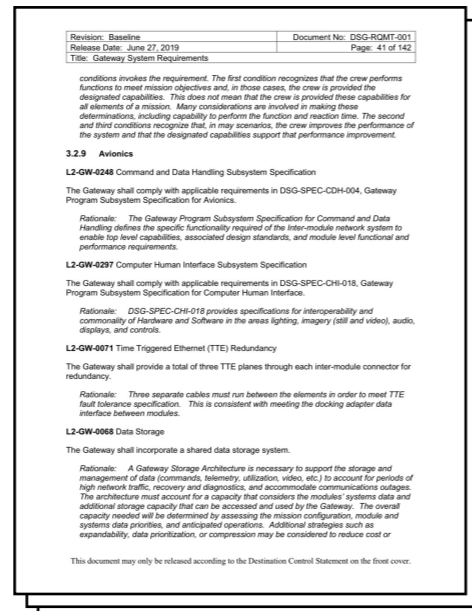


**How to detect and diagnose “stuff”?**

# Sam the Satellite Engineer



**Sam**



**[4]**

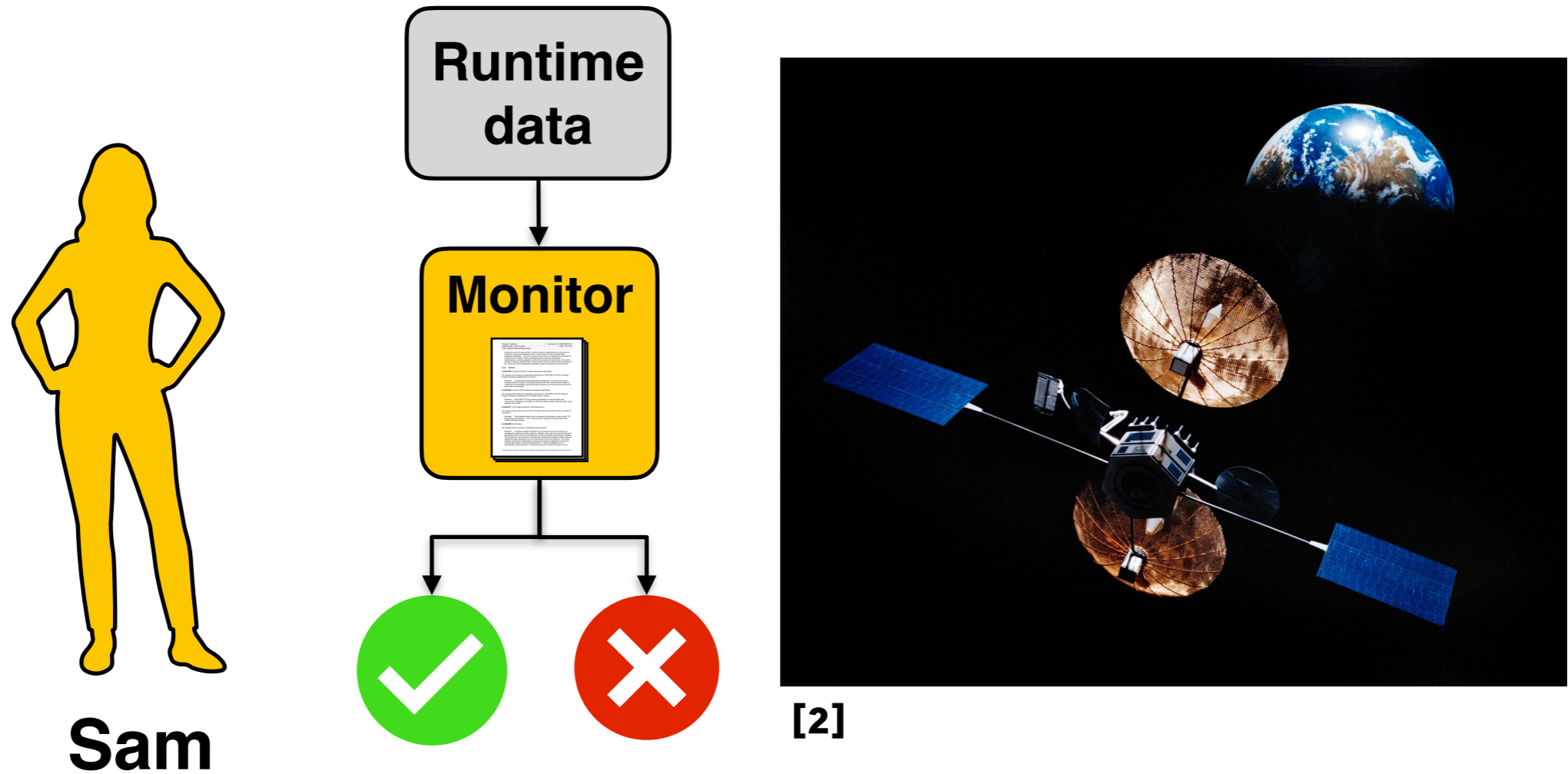


**[2]**

## How to detect and diagnose “stuff”?

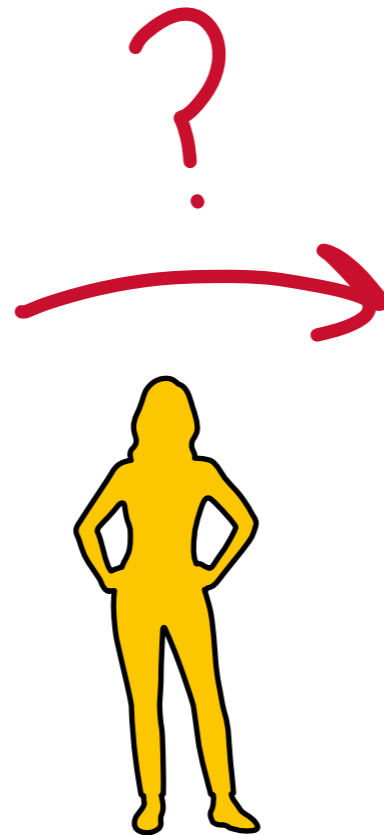
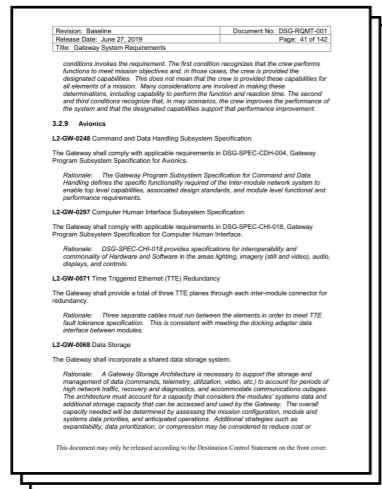
# Sam the Satellite Engineer

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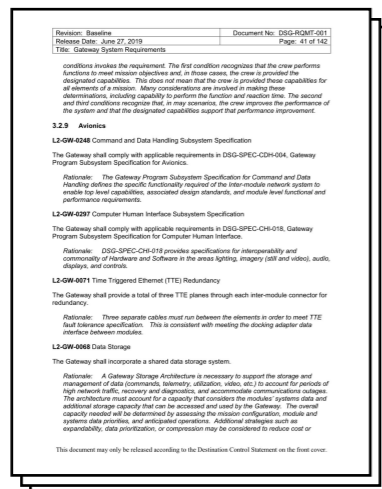


**How to detect and diagnose “stuff”?**

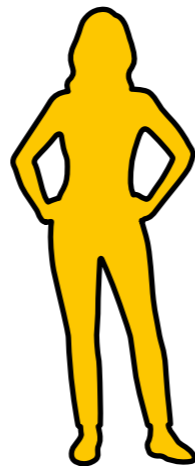
# How does Sam implement a monitor?



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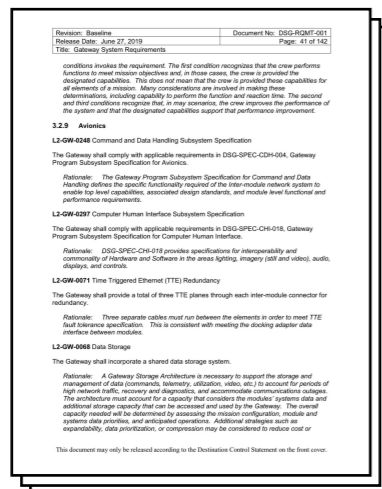
Code



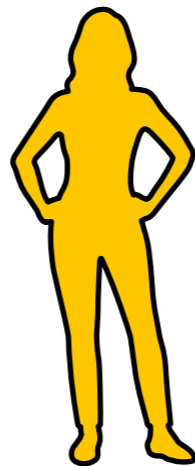
## Monitor

```
if(a && b) {  
  if(!c || d) {  
    return error_0;  
  } else if(c || d) {  
    if (e && f && g) {  
      return error_1;  
    } else if(e && h && g) {  
      return error_2;  
    } else {  
      return okay;  
    }  
  } else if(d) {  
    return error_1;  
  } else {  
    if(i && !j) {  
      return error_4;  
    } else if(!i && j) {  
      return error_5;  
    } else {  
      if((k || l) && (k == m)) {  
        return error_6;  
      } else {  
        return okay;  
      }  
    }  
  }  
}
```

# How does Sam implement a monitor?



Code

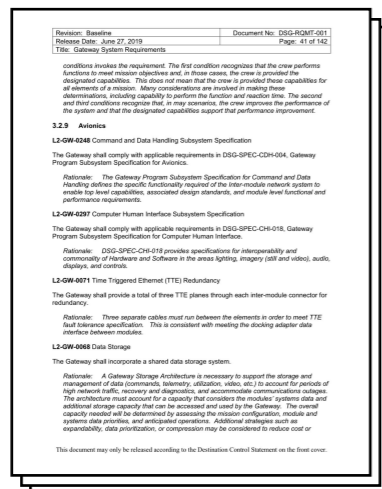


## Monitor

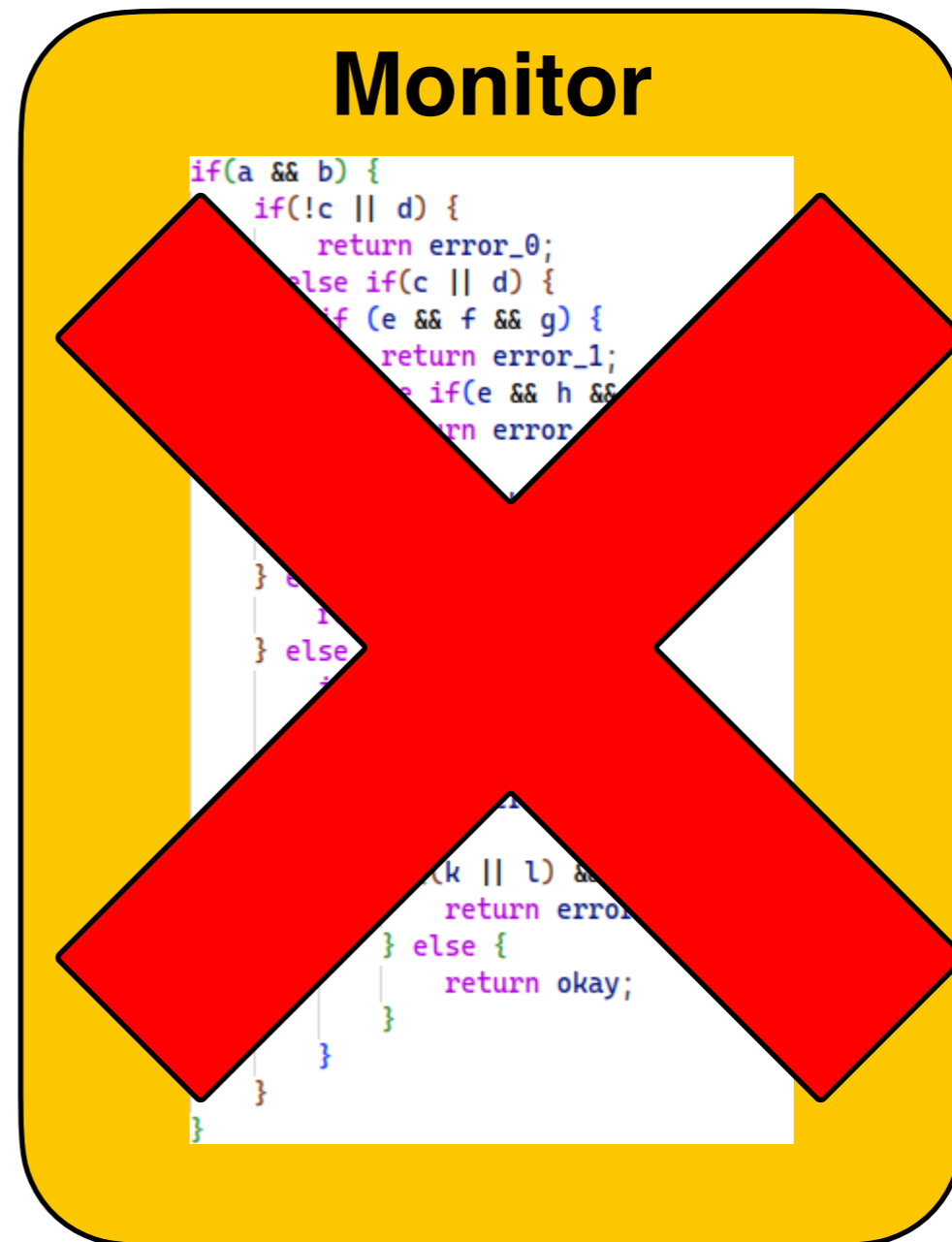
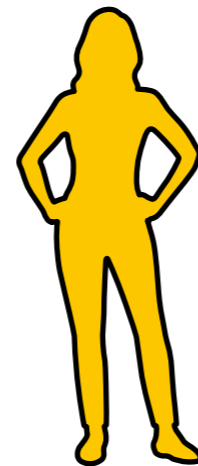
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        }  
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```

# How does Sam implement a monitor?

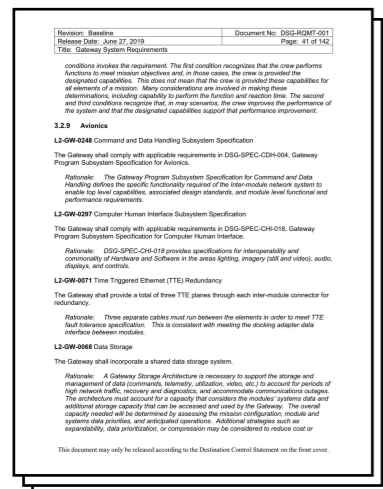
- > Re-certification
- > Opaque



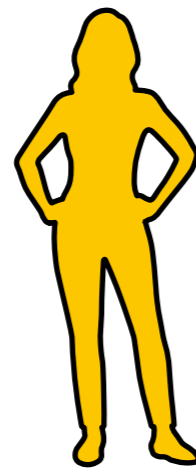
Code



# Runtime Verification (RV) Approach



**Formalize**



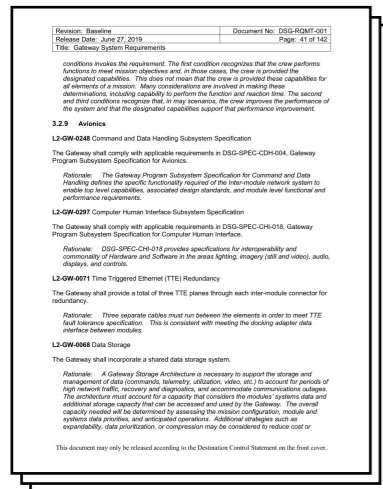
- > Temporal Logic
- > Automata
- > LOLA [5]

**Generate**

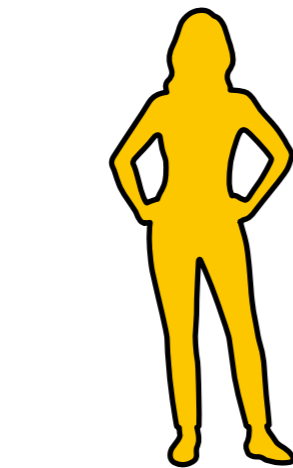


# Runtime Verification (RV) Approach

## What to use?



**Formalize**



- > Temporal Logic
- > Automata
- > LOLA [5]

**Generate**

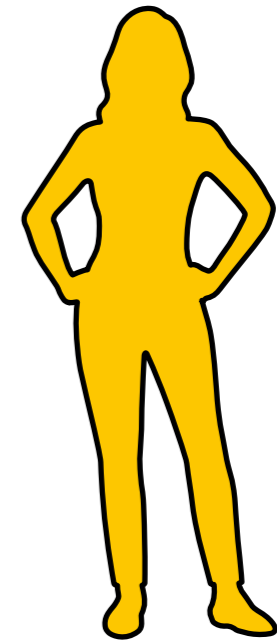
**Monitor**

# Which RV tool to use?

---

Sam's constraints (**embedded, real-time, safety-critical**):

- > Realizable
- > Responsive
- > Unobtrusive



# Which RV tool to use?

---

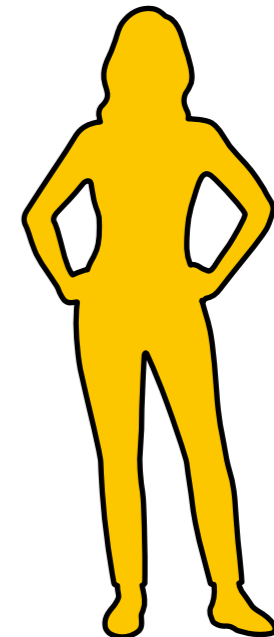
Sam's constraints (**embedded, real-time, safety-critical**):

- > **Realizable**

- > Use **bounded, little** resources
- > Adapt to new specifications **without re-compilation**
- > **Expressive, intuitive** specification language

- > **Responsive**

- > **Unobtrusive**



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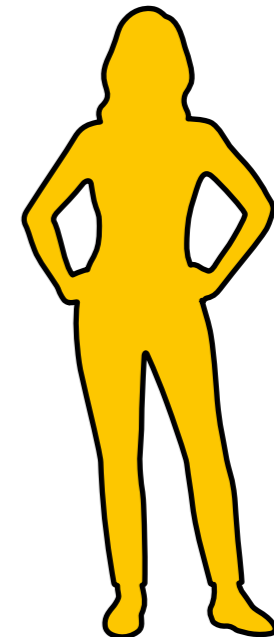
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- > **Responsive**

- > Runs continuously
- > Provides results **as soon as available**

- > **Unobtrusive**



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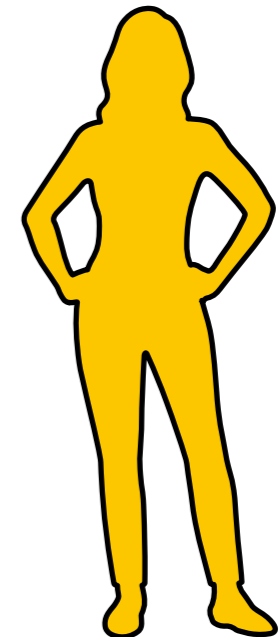


- > **Responsive**

- > Runs continuously
- > Provides results **as soon as available**

- > **Unobtrusive**

- > Does not impact **certifiability/functionality/behavior**
- > Black box

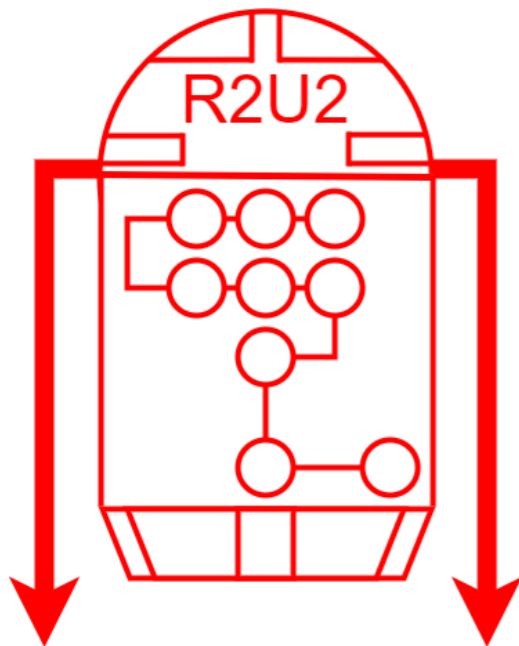


Perfect!

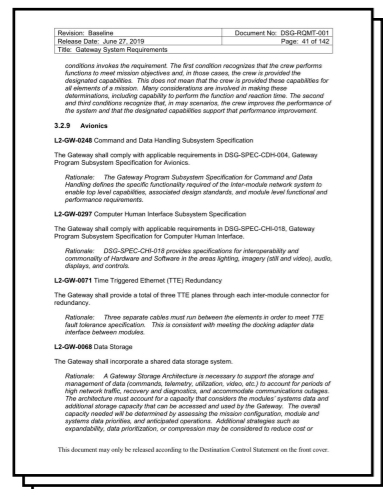
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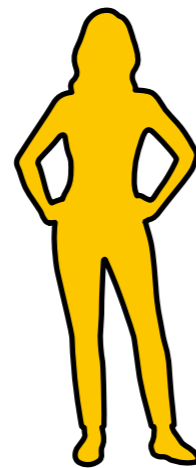
- > Realizable
- > Responsive
- > Unobtrusive
- > Unit



# R2U2 Approach



**Formalize**

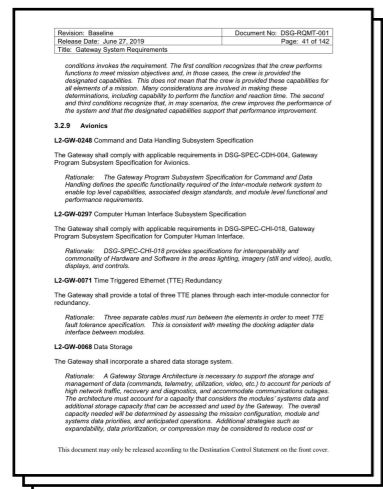


- > Temporal Logic
- > Automata
- > LOLA [5]

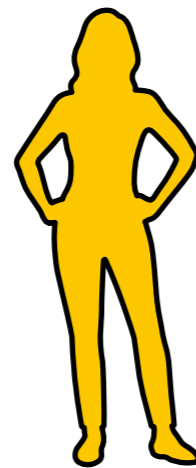
**Generate**



# R2U2 Approach



**Formalize**



**Mission-time LTL**

**Generate**



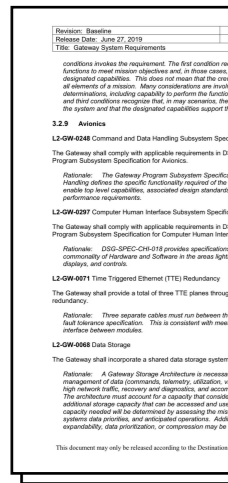
**Monitor**

# R2U2 Approach

## Mission-time LTL (MLTL) [16]

“If ‘task’ is in the scheduler, ‘task’ shall execute within 3 seconds ”

$$(sched_{task}) \rightarrow (\Diamond_{[0,3]} exec_{task})$$

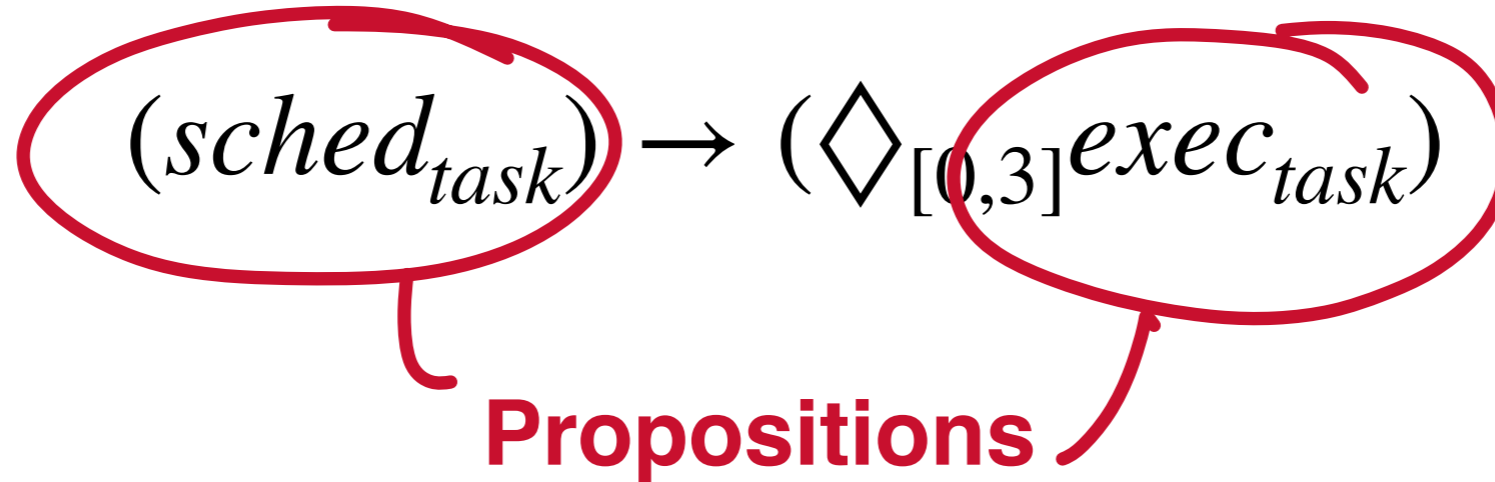


itor

# R2U2 Approach

## Mission-time LTL (MLTL) [16]

“If ‘task’ is in the scheduler, ‘task’ shall execute within 3 seconds ”



Revision: Baseline  
Released Date: June 27, 2019  
Title: Gateway System Requirements

conditions include the requirement. The first condition requires functions to meet mission objectives and, in those cases, designated capabilities. This does not mean that the crew will elements of a mission. Many considerations are made determinations, including capability to perform the function and their conditions recognize that, in any scenario, the system and that the designated capabilities support the

**3.2.9 Avionics**

**L2-GW-0248** Command and Data Handling Subsystem Specific  
The Gateway shall comply with applicable requirements in DS Program Subsystem Specification for Avionics.  
Rationale: The Gateway Program Subsystem Specification Handling defines the specific functionality required of the enable top level capabilities, associated design standards, performance requirements.

**L2-GW-0297** Computer Human Interface Subsystem Specific  
The Gateway shall comply with applicable requirements in DS Program Subsystem Specification for Computer Human Interface.  
Rationale: DS2-SPEC-CH-18 provides specifications commonality of Hardware and Software in the areas of input display, and control.

**L2-GW-0071** Time Triggered Ethernet (TTE) Redundancy  
The Gateway shall provide a total of three TTE planes through redundancy.  
Rationale: Three separate cables must run between the fault tolerance specification. This is consistent with most avionics between modules.

**L2-GW-0068** Data Storage  
The Gateway shall incorporate a shared data storage system.  
Rationale: A Gateway Storage Architecture is necessary management of data (commands, telemetry, utilization, and high network traffic, recovery and disposition, and access. The architecture must account for a capacity that consider additional storage capacity that can be accessed and used capacity needed will be determined by assessing the mission system data priorities, and anticipated operations. Avionics, expandability, data prioritization, or compression may be used.

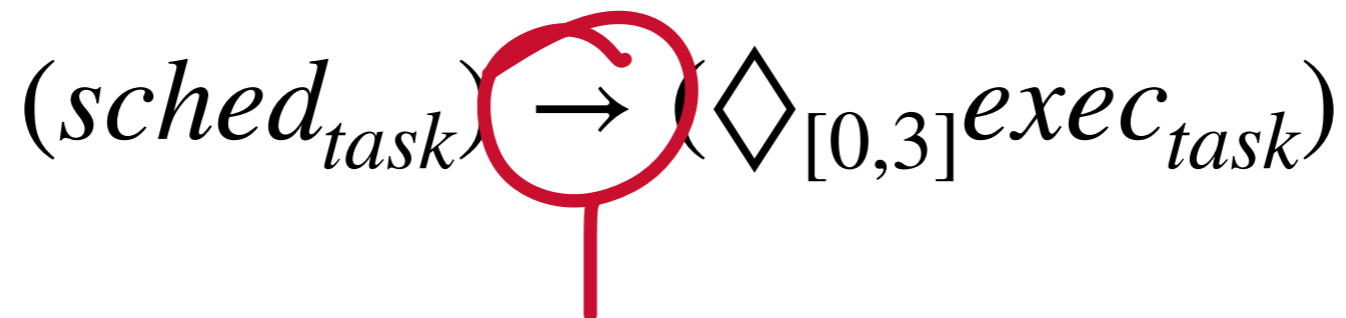
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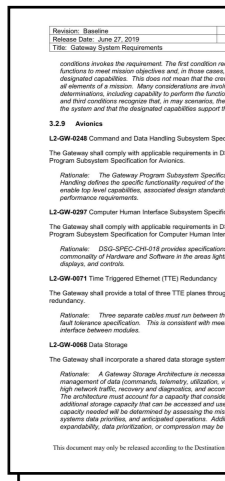
# R2U2 Approach

## Mission-time LTL (MLTL) [16]

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Logical connectives (  $\rightarrow$  ,  $\neg$  ,  $\wedge$  ,  $\vee$  )

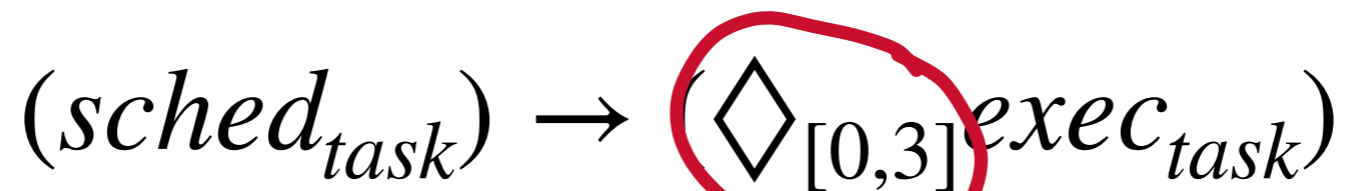


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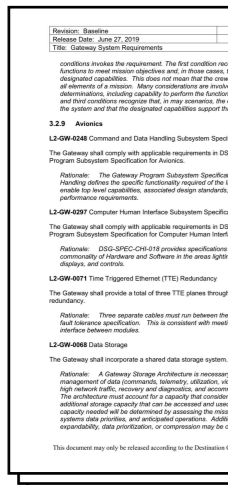
# R2U2 Approach

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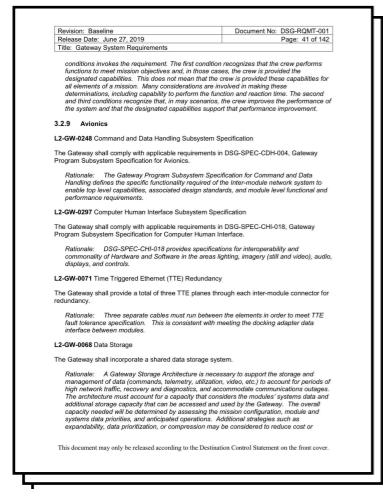


Future-time, bounded ops

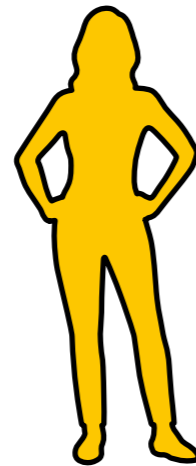


itor

# R2U2 Approach



**Formalize**



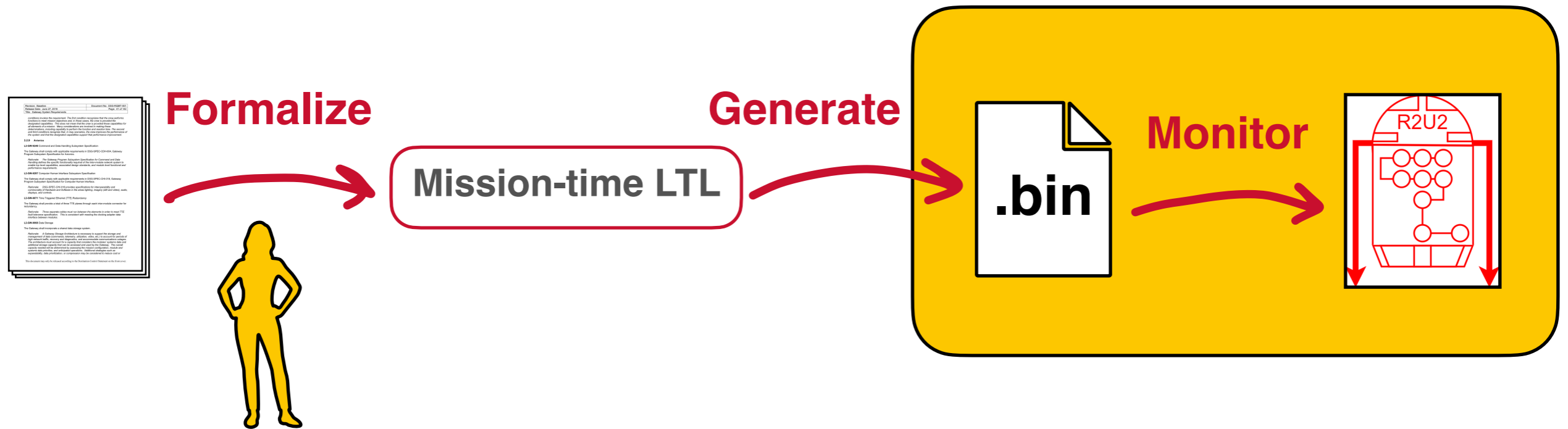
**Mission-time LTL**

**Generate**

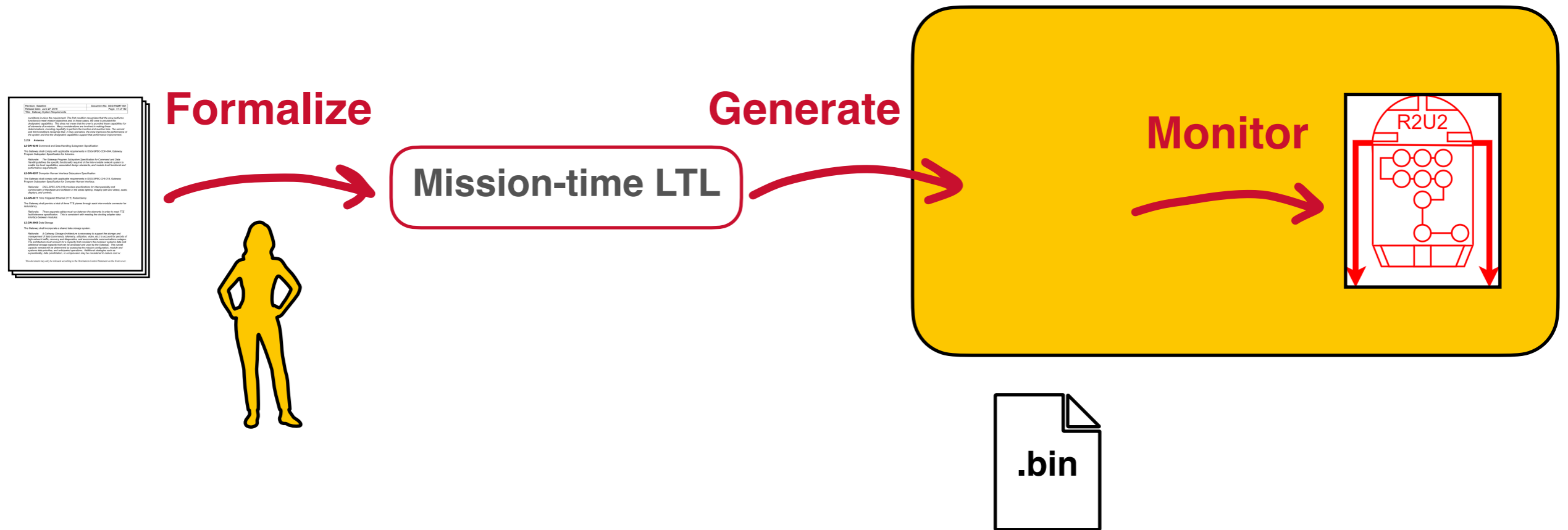


**Monitor**

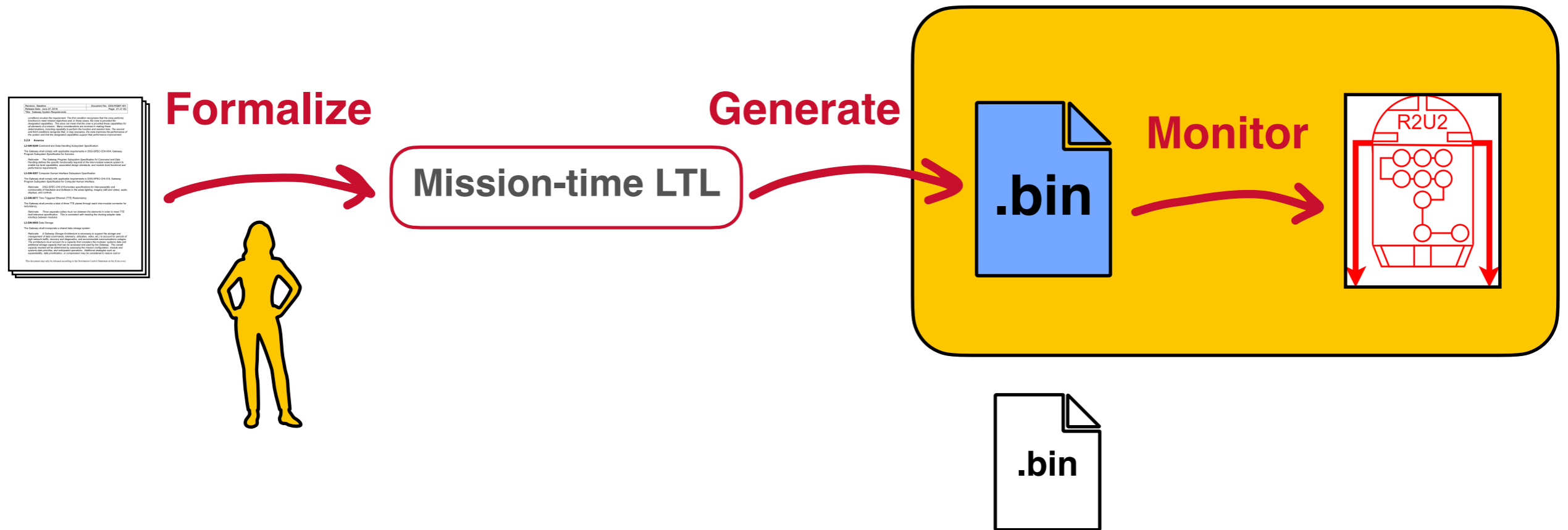
# R2U2 Approach



# R2U2 Approach

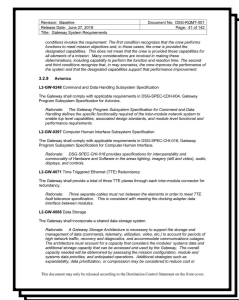


# R2U2 Approach

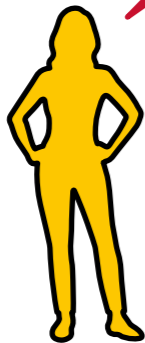


# R2U2 Approach

Formalizing is hard...



Formalize

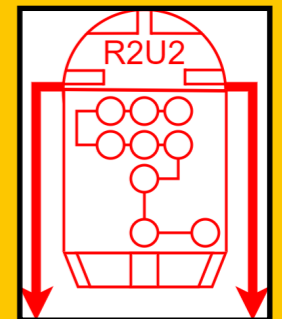


Mission-time LTL

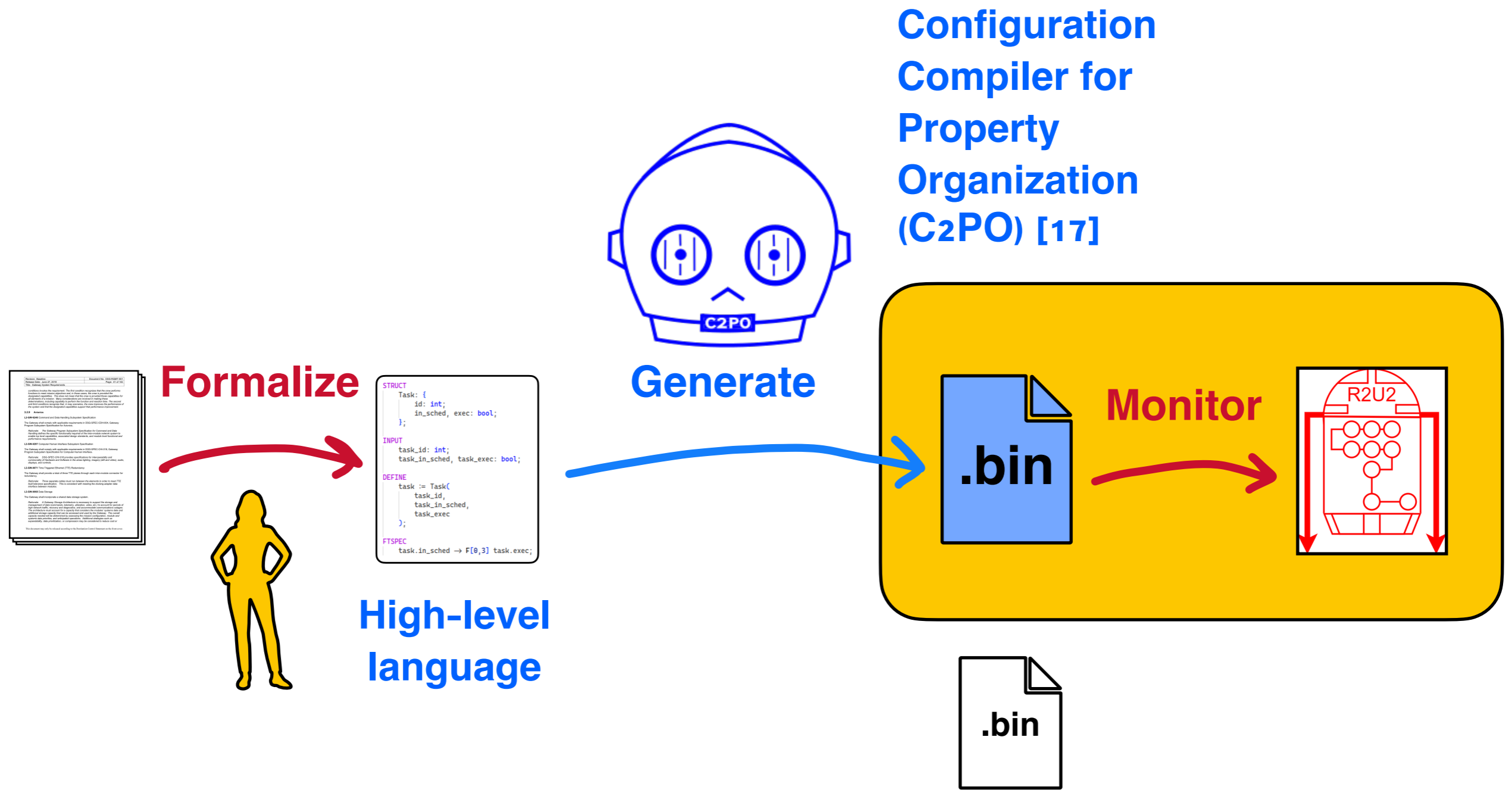
Generate



Monitor



# R2U2 + C2PO Approach



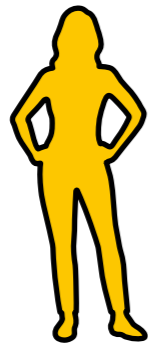
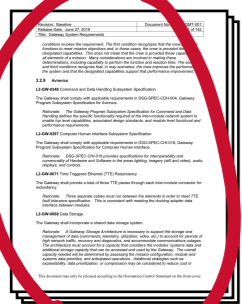
# R2U2 + C2PO Approach

Configuration  
Compiler for  
Property  
Organization  
(C2PO) [17]



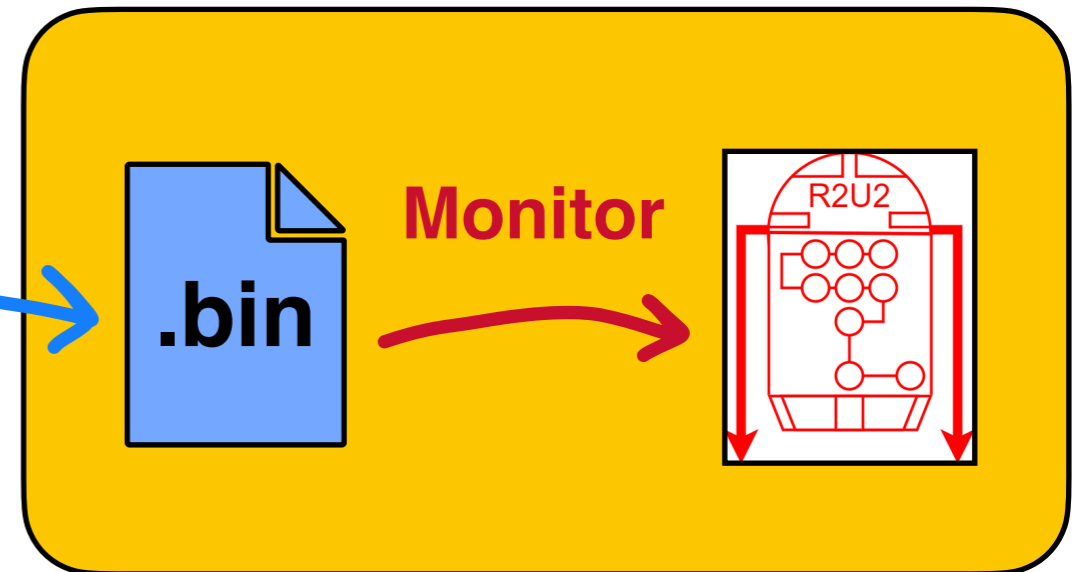
Generate

Formalize



```
STRUCT
  Task: {
    id: int;
    in_sched, exec: bool;
  };
INPUT
  task_id: int;
  task_in_sched, task_exec: bool;
DEFINE
  task := Task(
    task_id,
    task_in_sched,
    task_exec
  );
FTSPEC
  task_in_sched → F[0,3] task_exec;
```

High-level  
language



## C2PO Input Language [17]

---

“If ‘task’ is in the scheduler, ‘task’ shall execute within 3 seconds ”

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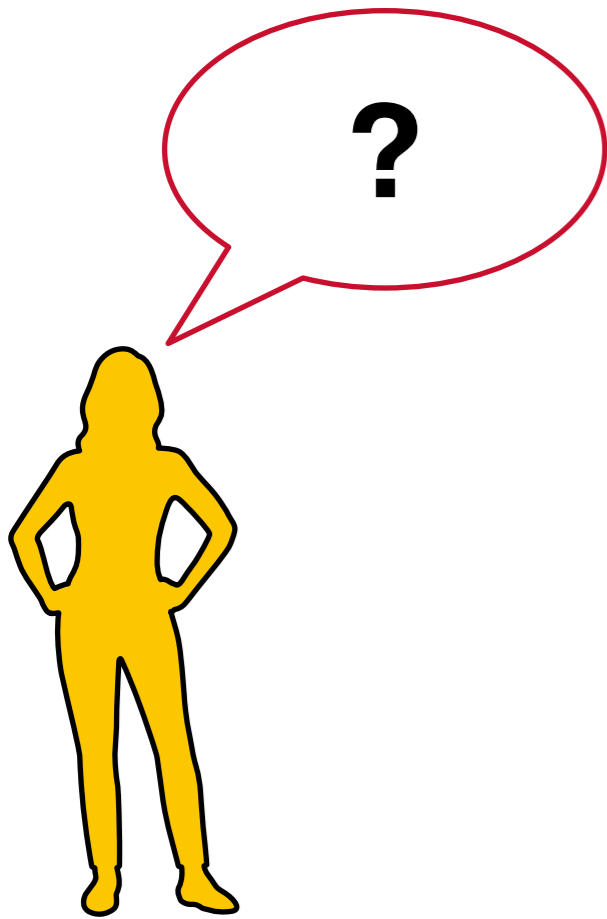
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## Sam's Specification Revisited

---

**“If ‘task’ is in the scheduler, ‘task’ shall execute within 3 seconds ”**



## Sam's Specification Revisited

---

**Every task in the scheduler**

~~“If ‘task’ is in the scheduler, ‘task’ shall execute within 3 seconds ”~~



**Aha!**

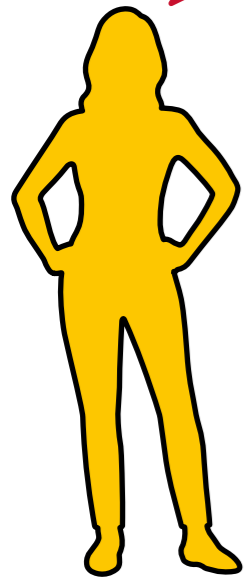
## Sam's Specification Revisited

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Formalize

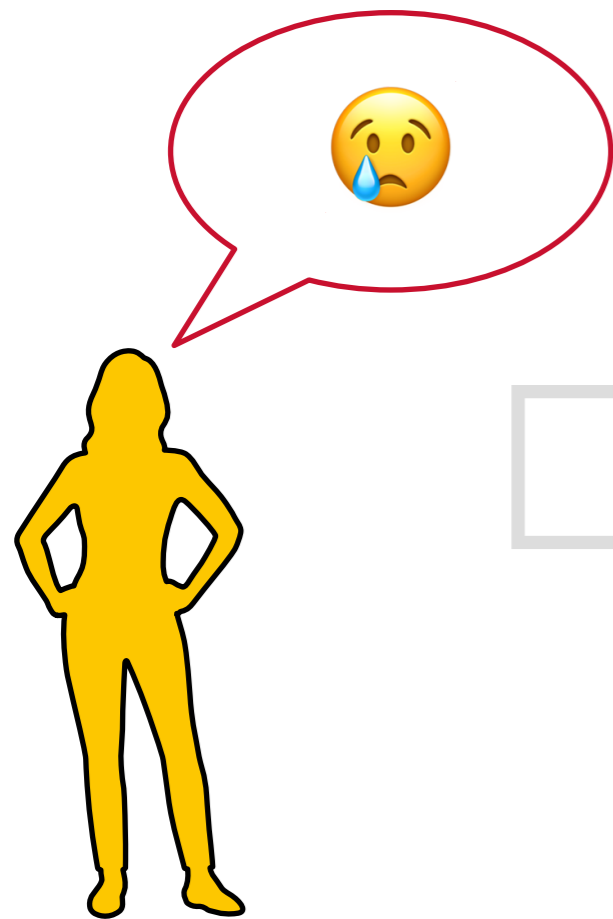
$$\square \forall t : Sched(t) \rightarrow \diamond_{[0,3]} Exec(t)$$

## Sam's Specification Revisited

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Formalize



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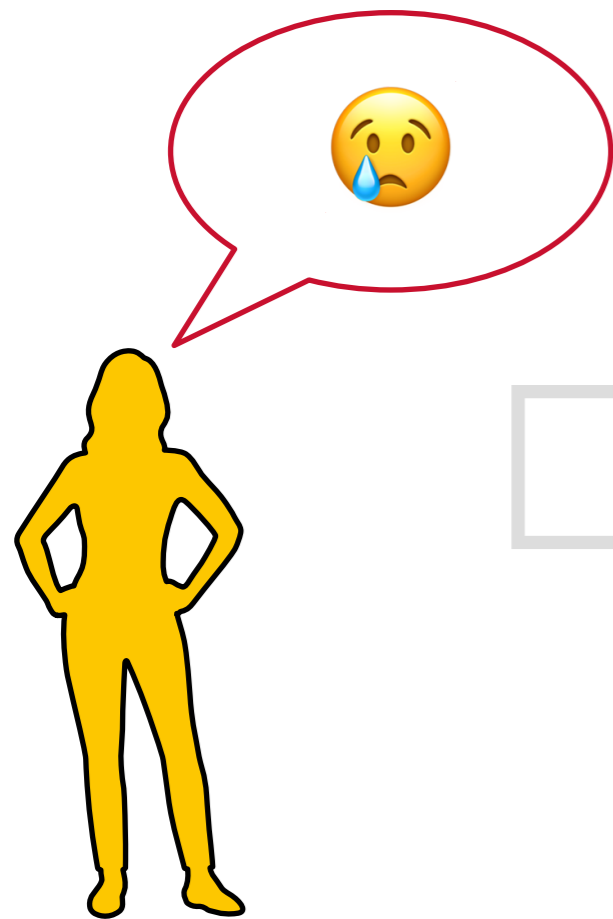
Can't use R2U2

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Formalize



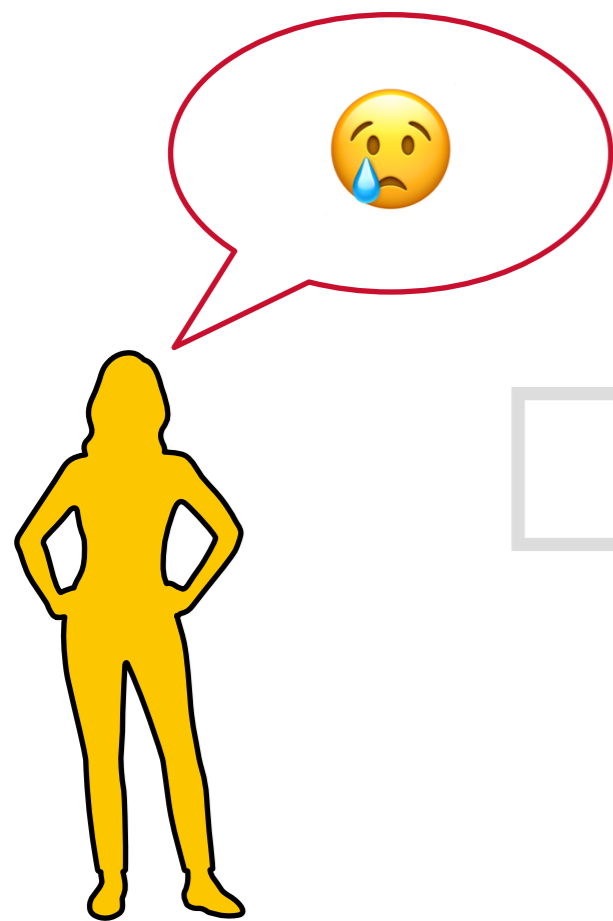
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Can't use R2U2 ...or can she?

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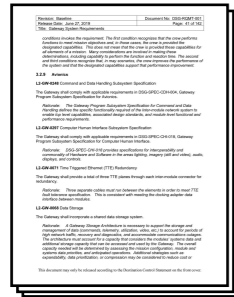
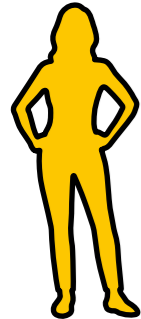


Formalize

~~$\forall t : Sched(t) \rightarrow \diamond_{[0,3]} Exec(t)$~~

Can't use R2U2 ...or can she?

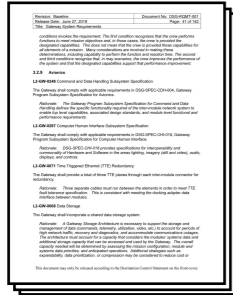
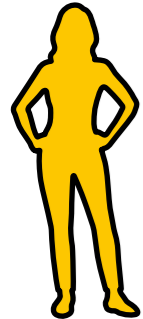
# Patterns in Sam's Requirements



- “Each request in the queue shall...”
- “All active sensors shall...”
- “Every task in the scheduler shall execute within 3 seconds”

$$\square \quad \forall t : Sched(t) \rightarrow \diamond_{[0,3]} Exec(t)$$

# Patterns in Sam's Requirements



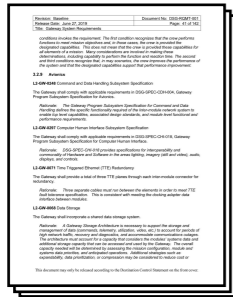
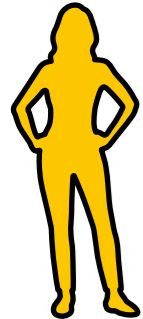
- “Each request in the queue shall...”
- “All active sensors shall...”
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$$\square \quad \forall t : Sched(t) \rightarrow \diamond_{[0,3]} Exec(t)$$

> **Guarded quantifiers:**

$$\forall x : Set(x) \rightarrow \Phi$$
$$\exists x : Set(x) \wedge \Phi$$

# Patterns in Sam's Requirements



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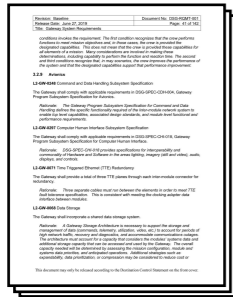
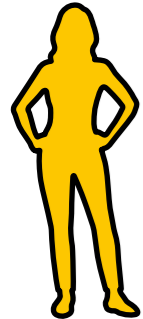
> **Guarded quantifiers:**

$$\forall x : Set(x) \rightarrow \Phi$$

$$\exists x : Set(x) \wedge \Phi$$

> **Bounded Dynamic Set:** A set that changes over time and has a bounded cardinality

# Patterns in Sam's Requirements



“Each request in the **queue** shall...”

“All **active sensors** shall...”

“Every task in the **scheduler** shall execute within 3 seconds”

$$\square \forall t : Sched(t) \rightarrow \diamond_{[0,3]} Exec(t)$$

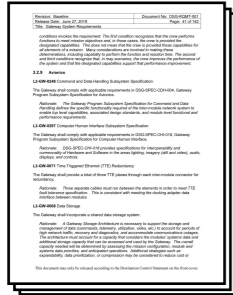
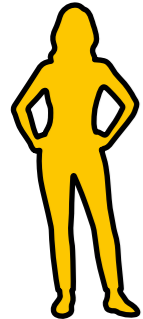
> **Guarded quantifiers:**

$$\forall x : Set(x) \rightarrow \Phi$$

$$\exists x : Set(x) \wedge \Phi$$

> **Bounded Dynamic Set:** A set that changes over time and has a bounded cardinality

# Patterns in Sam's Requirements



“Each request in the **queue** shall...” **Require**  
 “All **active sensors** shall...”  $|Sched| \leq \max(Sched)$   
 “Every task in the **scheduler** shall execute within 3 seconds”

$$\square \forall t : Sched(t) \rightarrow \diamond_{[0,3]} Exec(t)$$

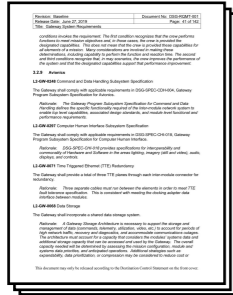
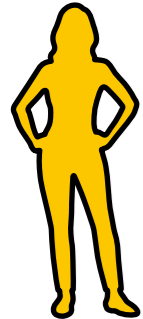
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# Patterns in Sam's Requirements



- “Each request in the **queue** shall...”
- “All **active sensors** shall...”
- “Every task in the **scheduler** shall execute within 3 seconds”

$$\square \forall t : Sched(t) \rightarrow \diamond_{[0,3]} Exec(t)$$

> **Guarded quantifiers:**

$$\forall x : Set(x) \rightarrow \Phi$$

$$\exists x : Set(x) \wedge \Phi$$

> **Bounded Dynamic Set:** A set that changes over time and has a bounded cardinality

= FO-MLTL

## FO-MLTL Quantifier Elimination

---

$$\cancel{\forall} t : \textit{Sched}(t) \longrightarrow \diamond_{[0,3]} \textit{Exec}(t)$$

## FO-MLTL Quantifier Elimination

---

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$\bigwedge_{i \in [0, \max(\text{Sched}) - 1]} (\text{Sched}(t_i) \rightarrow \diamond_{[0,3]} \text{Exec}(t_i))$$

# FO-MLTL Quantifier Elimination

---

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$\bigwedge_{i \in [0, \max(\text{Sched}) - 1]} (\text{Sched}(t_i) \rightarrow \diamond_{[0,3]} \text{Exec}(t_i))$$

$$\wedge |\text{Sched}| \leq \max(\text{Sched})$$

**Cardinality Constraint**

## FO-MLTL Quantifier Elimination

---

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$\bigwedge_{i \in [0, \max(\text{Sched}) - 1]} (\text{Sched}(t_i) \rightarrow \diamond_{[0,3]} \text{Exec}(t_i))$$

## FO-MLTL Quantifier Elimination

---

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$\text{max}(\text{Sched}) = 2$$

# FO-MLTL Quantifier Elimination

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

**Need to assign meaning**

# FO-MLTL Quantifier Elimination

---

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

**Sched:**

**A**

# FO-MLTL Quantifier Elimination

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

**Arbitrarily assign**

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

**Sched:**

**A**

# FO-MLTL Quantifier Elimination

---

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

**Sched:**

**A**

# FO-MLTL Quantifier Elimination

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

**Sched:**

**A**

# FO-MLTL Quantifier Elimination

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$$

$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

**Sched:**

**A**

# FO-MLTL Quantifier Elimination

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$$

$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

**Always False**

**Sched:**

**A**

# FO-MLTL Quantifier Elimination

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$$

$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

**Vacuously True**

**Sched:**

**A**

# FO-MLTL Quantifier Elimination

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$$

$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

**Vacuously True**

**Sched:**

**A**

**Changes over time**

## FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

	0	1	2	3	4
<i>Sched</i>					






## FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

	0	1	2	3	4
<i>Sched</i>					

## FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

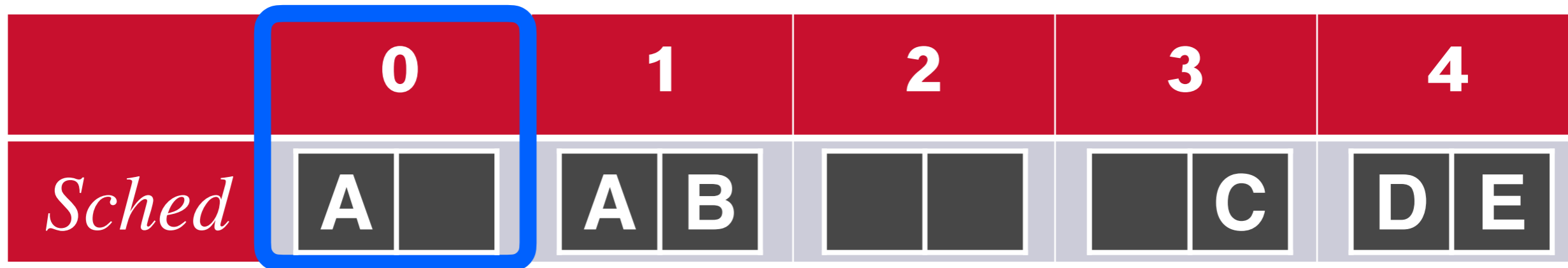
	0	1	2	3	4
<i>Sched</i>	A	A B		C	D E

# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge (\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

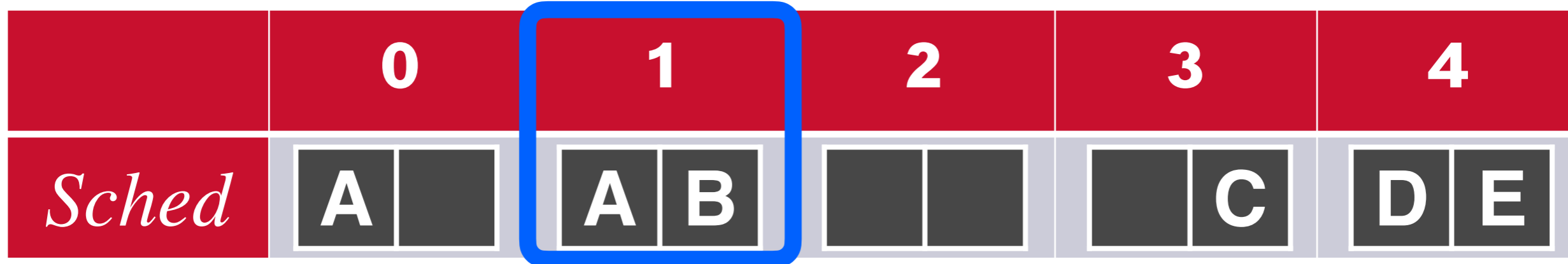


# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge (\text{Sched}(B) \rightarrow \diamond_{[0,3]} \text{Exec}(B))$$



# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp)) \wedge (\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

	0	1	2	3	4
<i>Sched</i>	A	A B		C	D E

# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

**Instantiate**

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$$
$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$$
$$(\text{Sched}(B) \rightarrow \diamond_{[0,3]} \text{Exec}(B))$$

$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp)) \wedge$$
$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

...

# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

**Instantiate**

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$$
$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$$
$$(\text{Sched}(B) \rightarrow \diamond_{[0,3]} \text{Exec}(B))$$

$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp)) \wedge$$
$$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

...

**MLTL Monitors (**  **)**

# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge$$

$$(\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

**Instantiate**

How many?

$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$   
 $(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$

$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge$   
 $(\text{Sched}(B) \rightarrow \diamond_{[0,3]} \text{Exec}(B))$

$(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp)) \wedge$   
 $(\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$

**MLTL Monitors ( .bin )**

# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

**Unroll** Will know answer by time 3

$$(\text{Sched}(A) \rightarrow \diamond_{[0,3]} \text{Exec}(A)) \wedge (\text{Sched}(\perp) \rightarrow \diamond_{[0,3]} \text{Exec}(\perp))$$

	0	1	2	3	4
<i>Sched</i>	A	A B		C	D E

# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \Diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(A) \rightarrow \Diamond_{[0,3]} \text{Exec}(A)) \wedge$$

$$(\text{Sched}(A) \rightarrow \Diamond_{[0,3]} \text{Exec}(A)) \wedge$$

$$(\text{Sched}(\perp) \rightarrow \Diamond_{[0,3]} \text{Exec}(\perp))$$

$$(\text{Sched}(A) \rightarrow \Diamond_{[0,3]} \text{Exec}(A)) \wedge$$

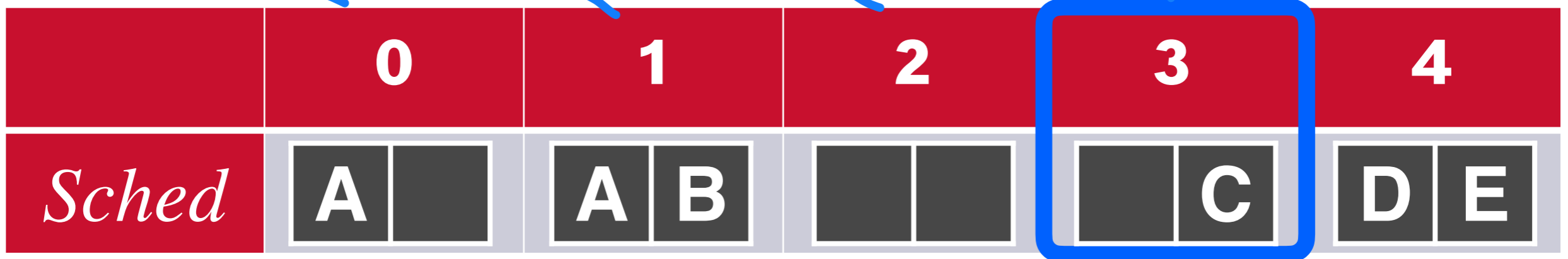
$$(\text{Sched}(B) \rightarrow \Diamond_{[0,3]} \text{Exec}(B))$$

$$(\text{Sched}(\perp) \rightarrow \Diamond_{[0,3]} \text{Exec}(\perp)) \wedge$$

$$(\text{Sched}(\perp) \rightarrow \Diamond_{[0,3]} \text{Exec}(\perp))$$

$$(\text{Sched}(\perp) \rightarrow \Diamond_{[0,3]} \text{Exec}(\perp)) \wedge$$

$$(\text{Sched}(C) \rightarrow \Diamond_{[0,3]} \text{Exec}(C))$$



# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \Diamond_{[0,3]} \text{Exec}(t)$$

**Unroll**

$$(\text{Sched}(A) \rightarrow \Diamond_{[0,3]} \text{Exec}(A)) \wedge$$

$$(\text{Sched}(A) \rightarrow \Diamond_{[0,3]} \text{Exec}(A)) \wedge$$

$$(\text{Sched}(\perp) \rightarrow \Diamond_{[0,3]} \text{Exec}(\perp))$$

$$(\text{Sched}(A) \rightarrow \Diamond_{[0,3]} \text{Exec}(A)) \wedge$$

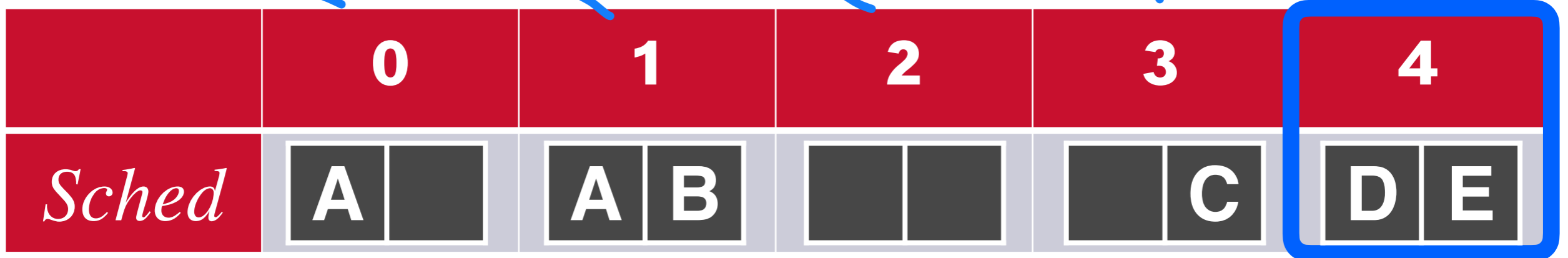
$$(\text{Sched}(B) \rightarrow \Diamond_{[0,3]} \text{Exec}(B))$$

$$(\text{Sched}(\perp) \rightarrow \Diamond_{[0,3]} \text{Exec}(\perp)) \wedge$$

$$(\text{Sched}(\perp) \rightarrow \Diamond_{[0,3]} \text{Exec}(\perp))$$

$$(\text{Sched}(\perp) \rightarrow \Diamond_{[0,3]} \text{Exec}(\perp)) \wedge$$

$$(\text{Sched}(C) \rightarrow \Diamond_{[0,3]} \text{Exec}(C))$$

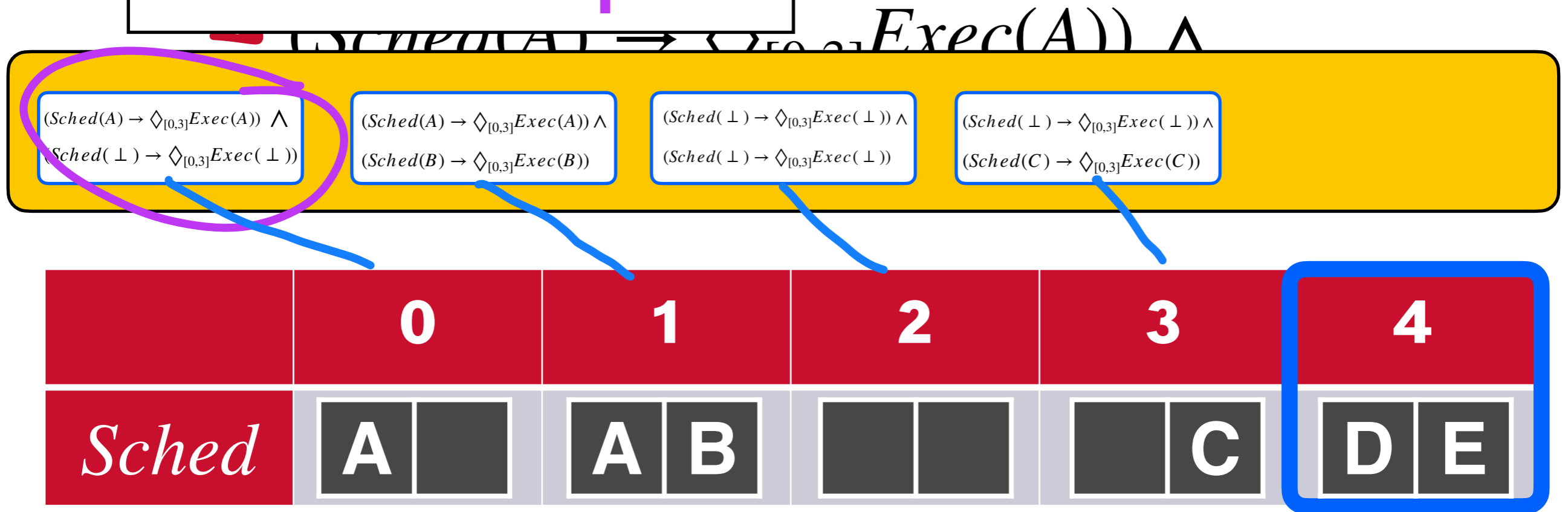


# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

Unroll

Re-use this space!



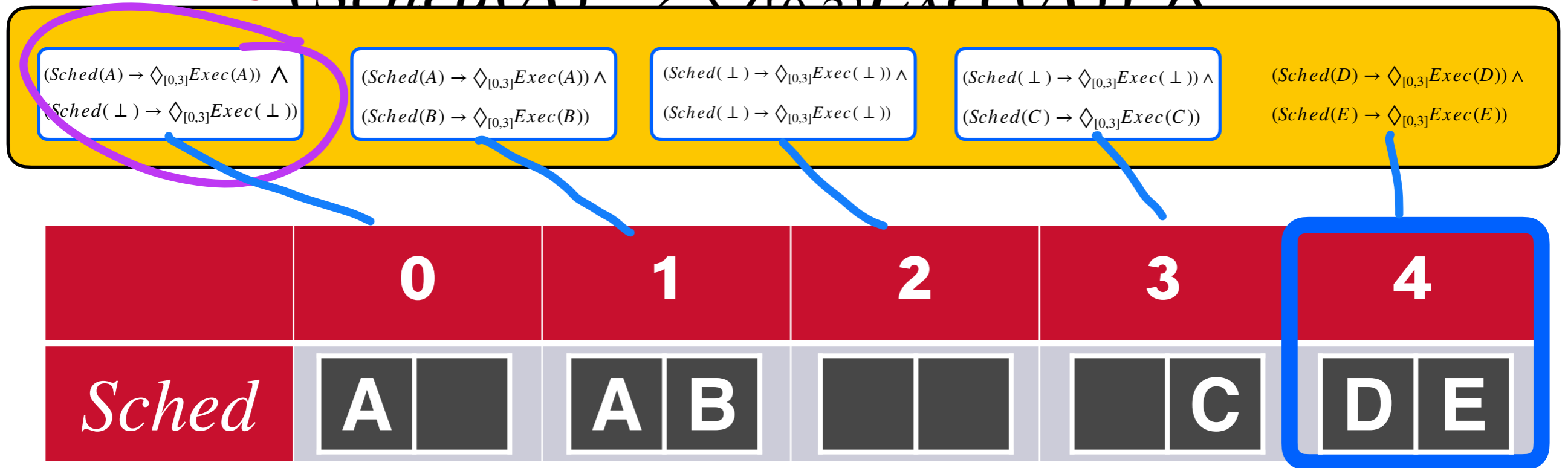
# FO-MLTL Quantifier Elimination

$$\forall t: \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

Unroll

Re-use this space!

Only need space for 4 formulas



# FO-MLTL Quantifier Elimination

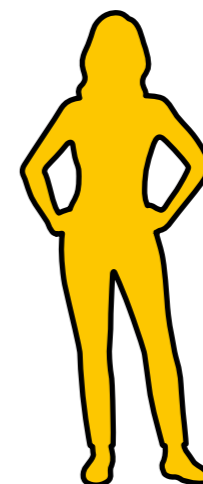
$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$



# FO-MLTL Quantifier Elimination

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

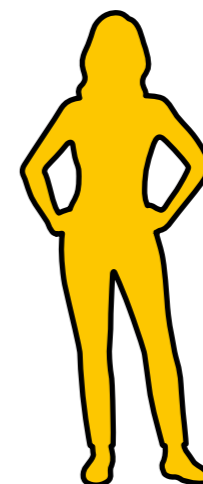
$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

**(Quantifier-free) MLTL formulas**



# FO-MLTL Quantifier Elimination

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

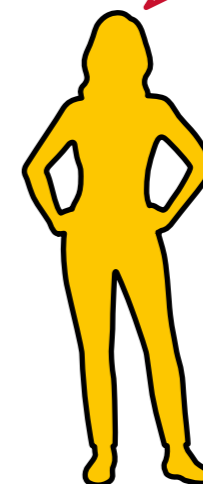
$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

Yay!

(Quantifier-free) MLTL formulas

Can monitor using R2U2!



# FO-MLTL Quantifier Elimination

$$\forall t : \text{Sched}(t) \rightarrow \diamond_{[0,3]} \text{Exec}(t)$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

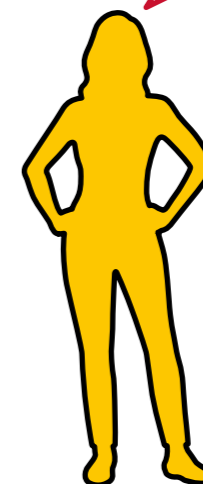
$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

$$(\text{Sched}(t_0) \rightarrow \diamond_{[0,3]} \text{Exec}(t_0)) \wedge (\text{Sched}(t_1) \rightarrow \diamond_{[0,3]} \text{Exec}(t_1))$$

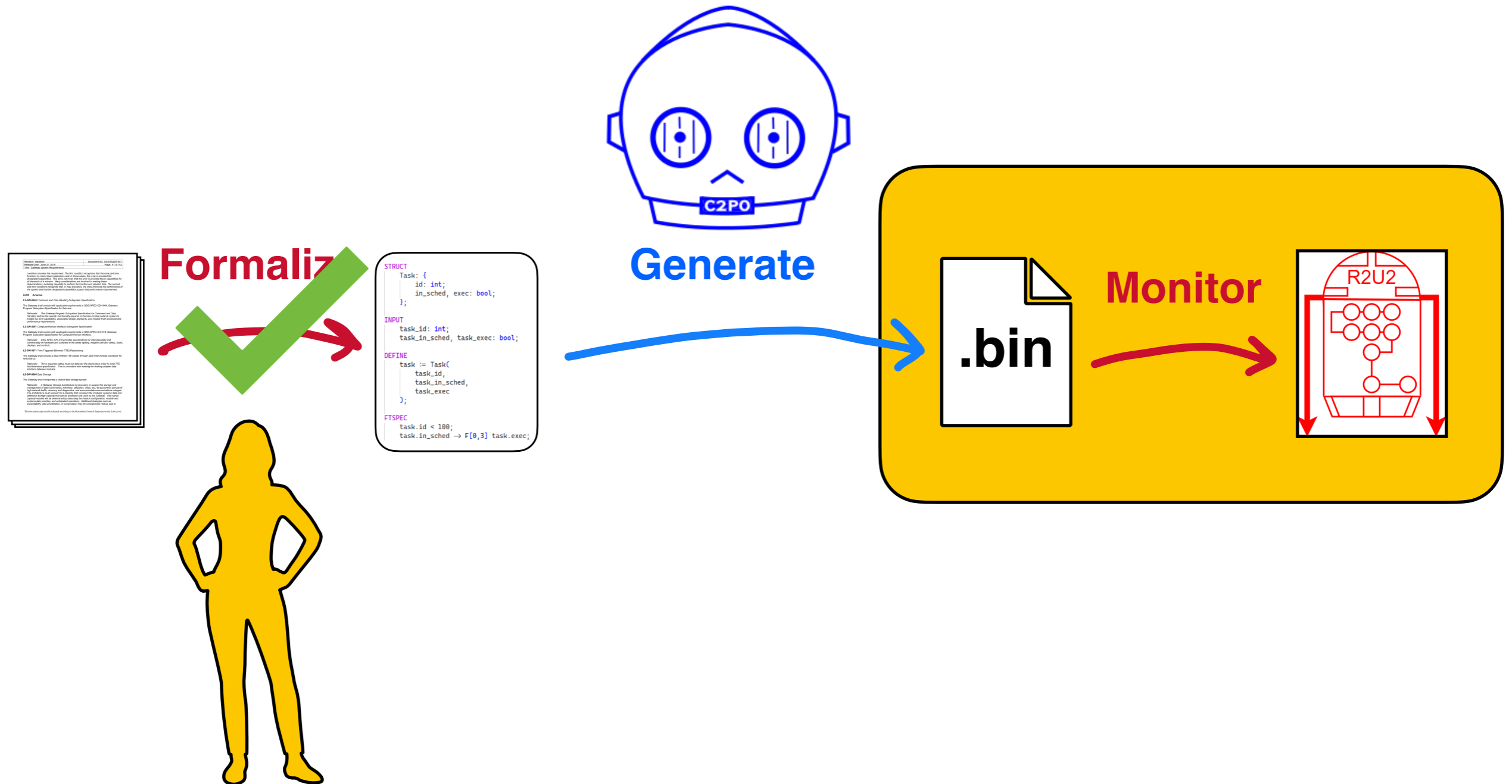
Yay!

(Quantifier-free) MLTL formulas

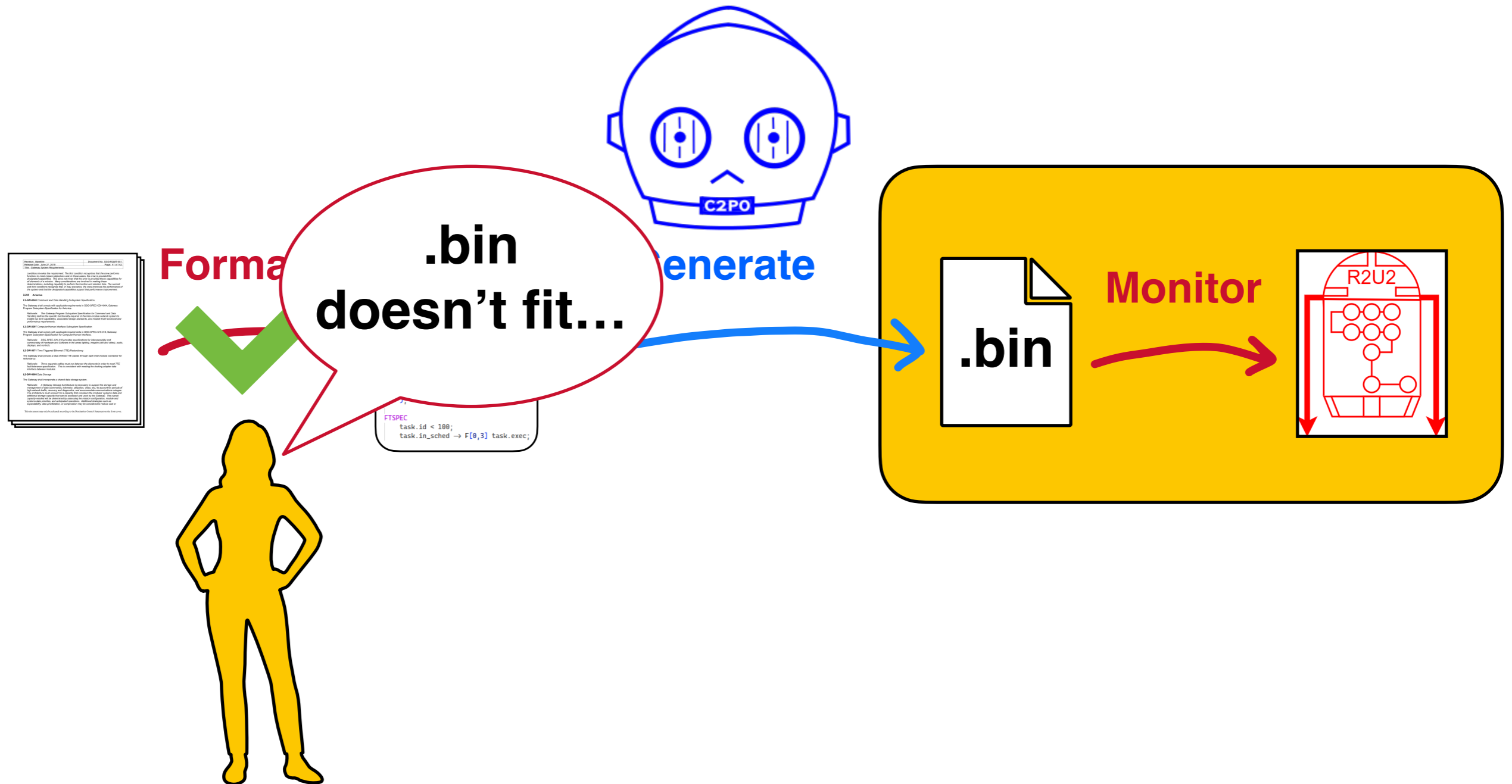
Can monitor using R2U2!\*\*\*



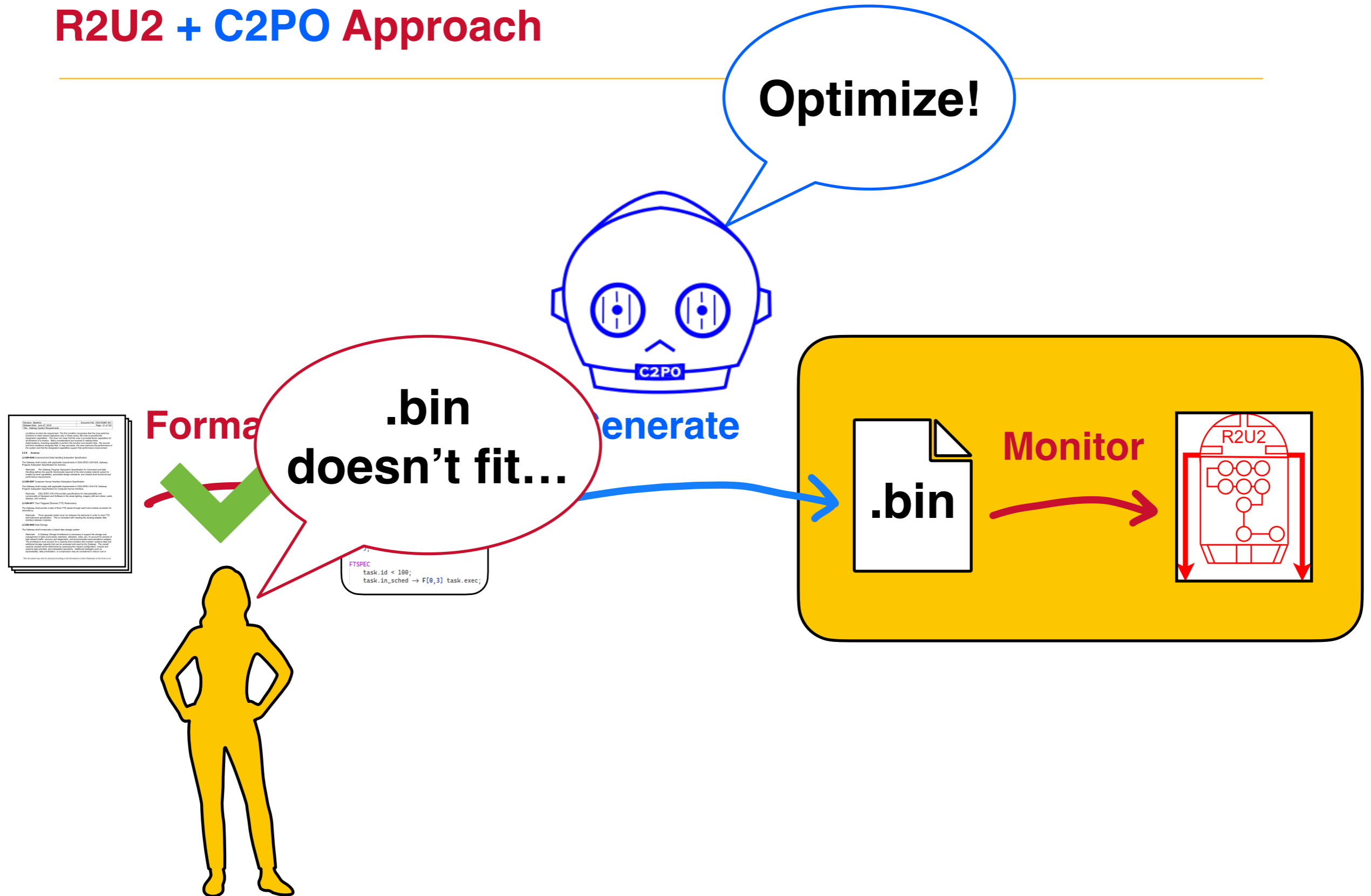
# R2U2 + C2PO Approach



# R2U2 + C2PO Approach

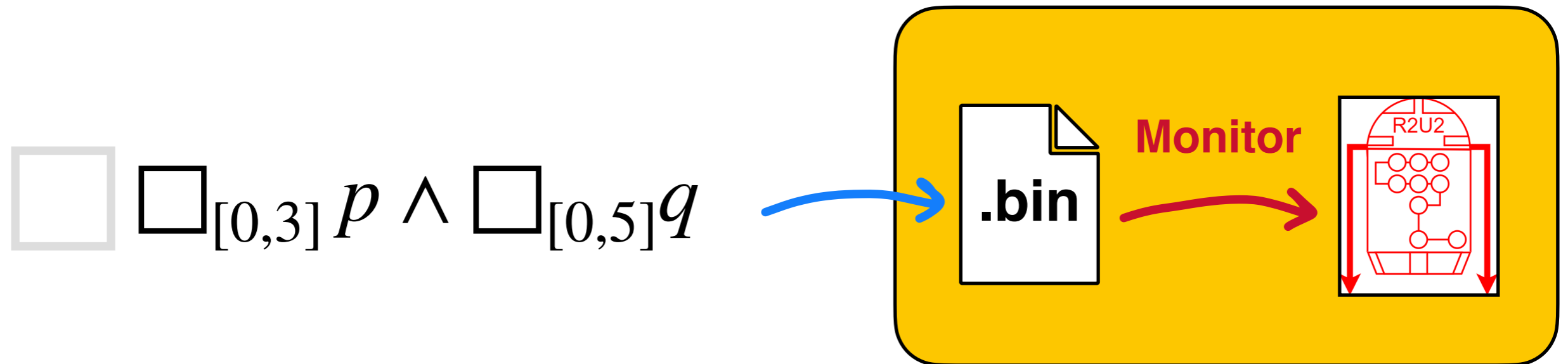


# R2U2 + C2PO Approach

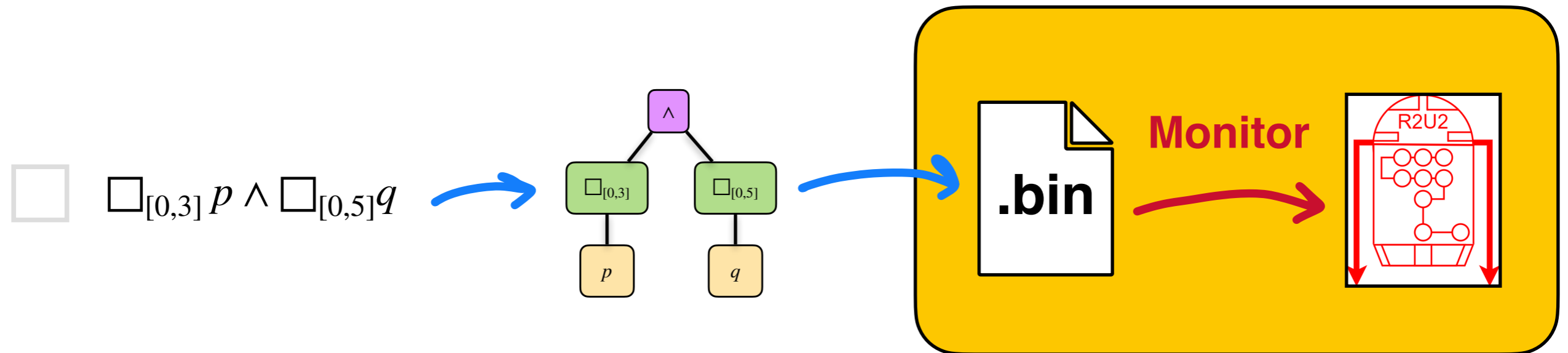


# How does R2U2 Encode MLTL? [18]

---

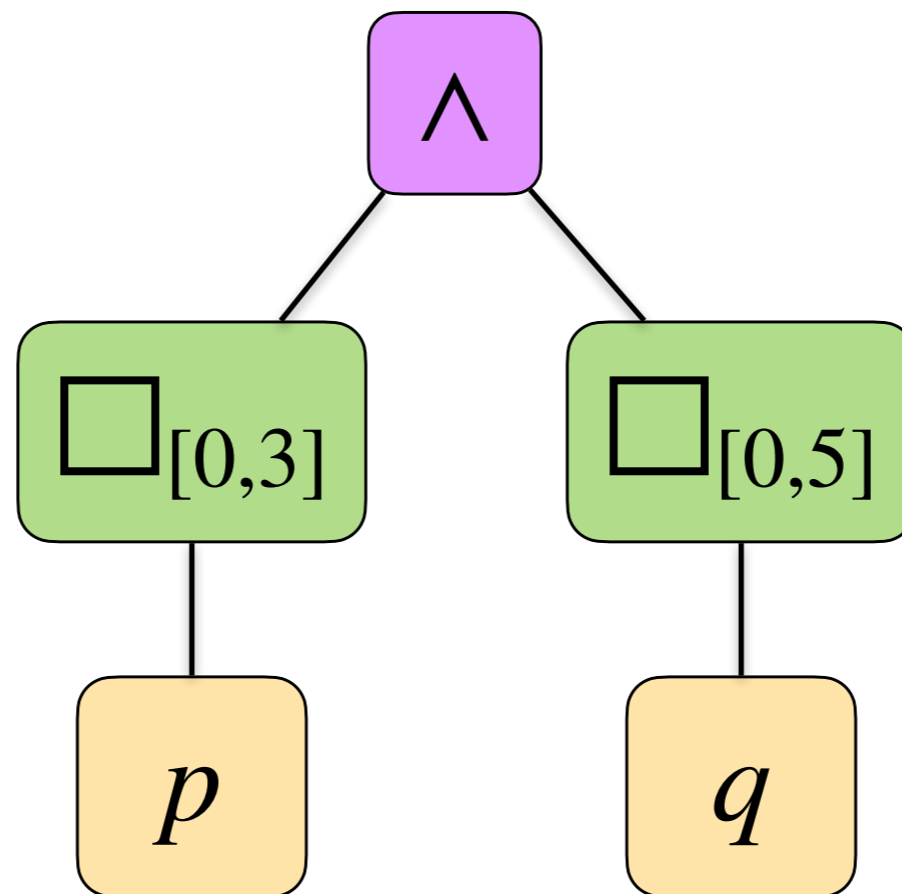


# How does R2U2 Encode MLTL? [18]



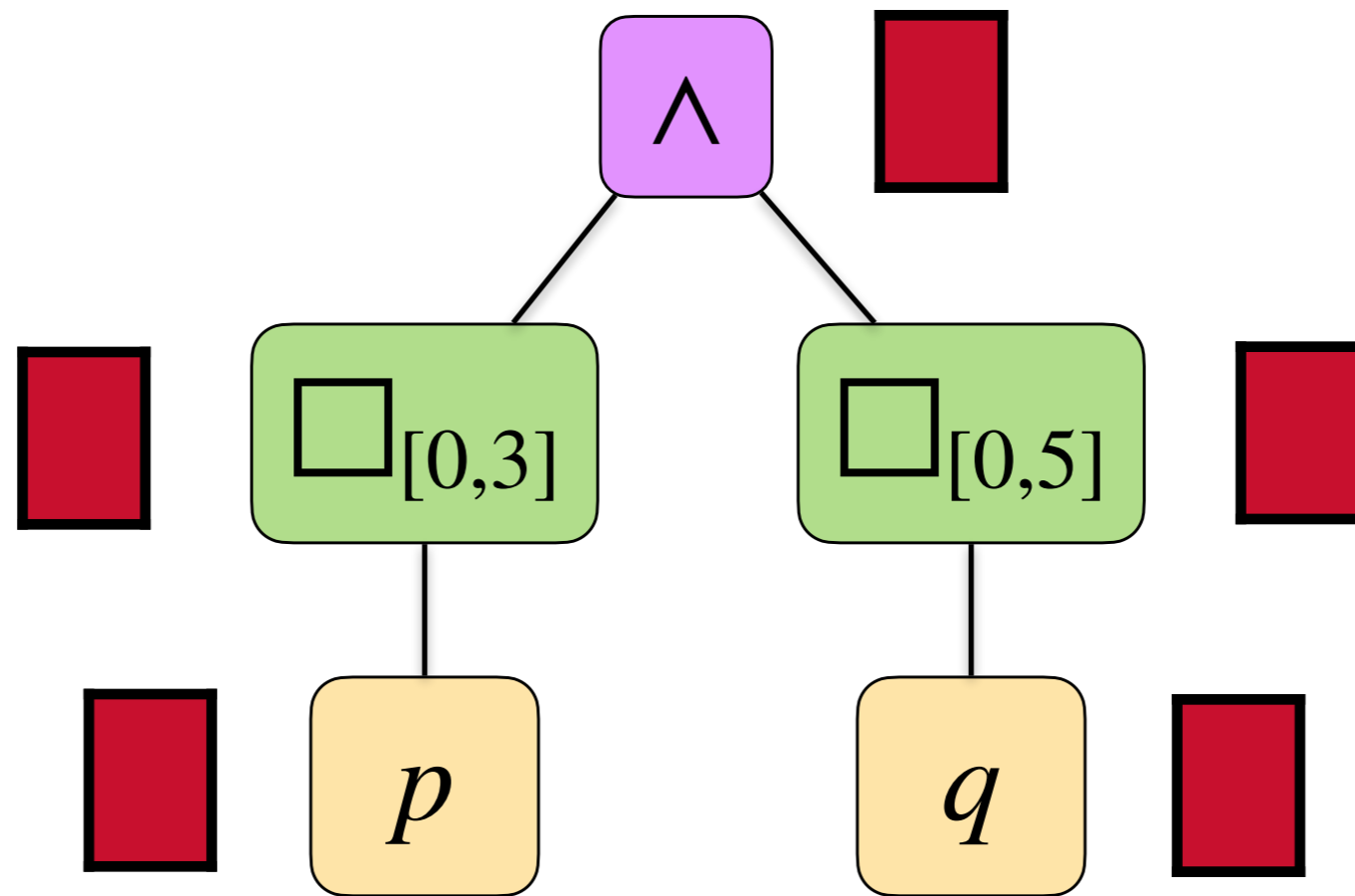
# How does R2U2 Encode MLTL? [18]

---



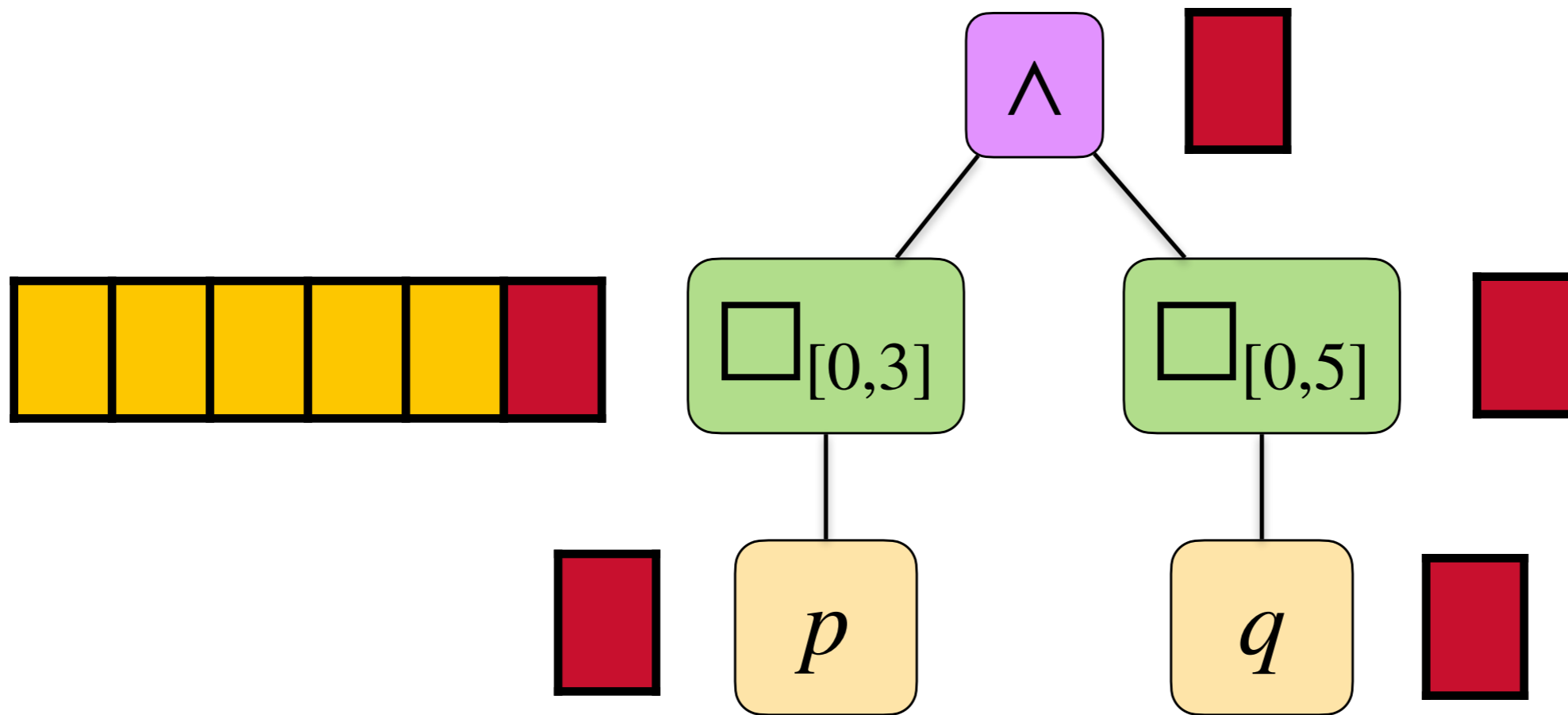
# How does R2U2 Encode MLTL? [18]

---



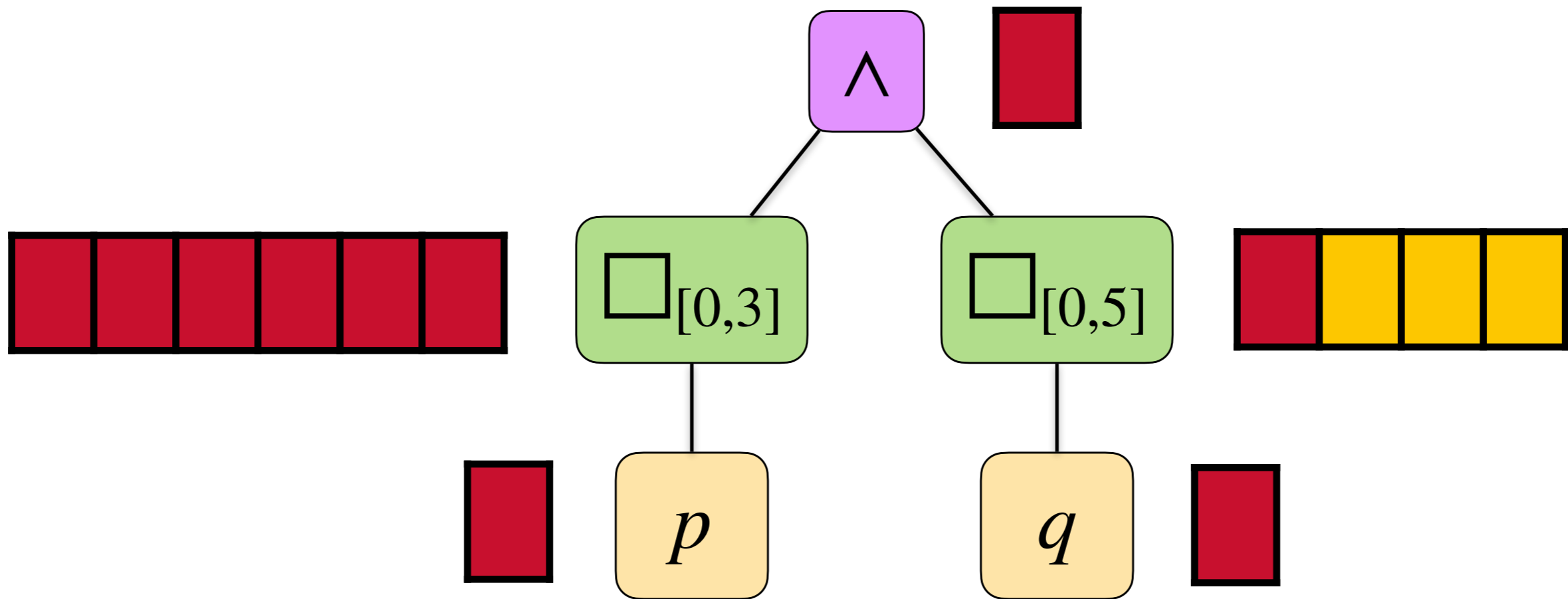
# How does R2U2 Encode MLTL? [18]

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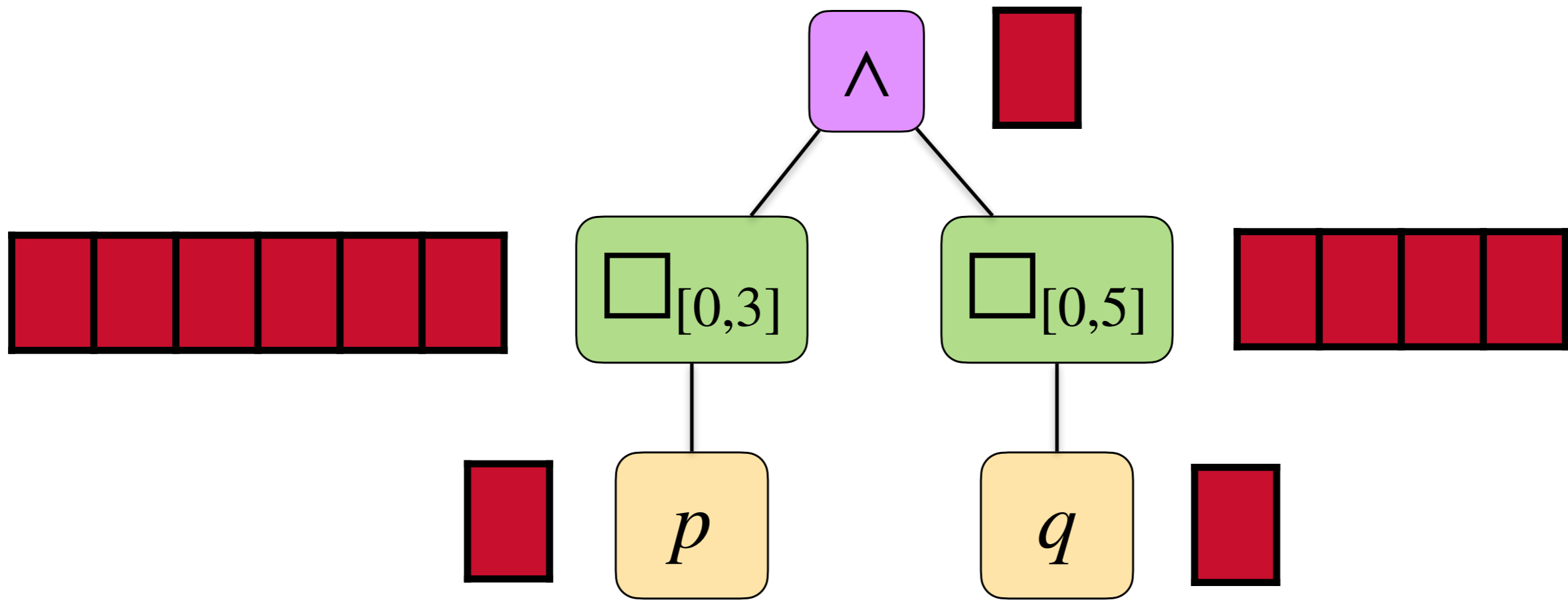


# How does R2U2 Encode MLTL? [18]

---



# How does R2U2 Encode MLTL? [18]



**| buffer | ~ upper bounds of siblings**

- > **Reduce number of nodes**
- > **Reduce sum of upper bounds**

# MLTL Rewrite Rules

---

$$\square_{[0, 3]} p \wedge \square_{[0, 5]} q$$

# MLTL Rewrite Rules

---

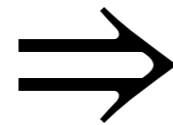
$$\square_{[0, 3]} p \wedge \square_{[0, 5]} q$$

Share the interval [0,3]

# MLTL Rewrite Rules

---

$$\square_{[0, 3]} p \wedge \square_{[0, 5]} q$$



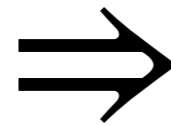
$$\square_{[0, 3]} (\square_{[0, 0]} p \wedge \square_{[0, 2]} q)$$

**Factor out common interval**

# MLTL Rewrite Rules

---

$$\square_{[0,3]} p \wedge \square_{[0,5]} q$$



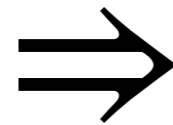
$$\square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q)$$

$$\square_{[0,0]} p \equiv p$$

# MLTL Rewrite Rules

---

$$\square_{[0, 3]} p \wedge \square_{[0, 5]} q$$



$$\square_{[0, 3]} (p \wedge \square_{[0, 2]} q)$$

# MLTL Rewrite Rules

---

$$\square_{[0, 3]} p \wedge \square_{[0, 5]} q$$

≡

$$\square_{[0, 3]} (p \wedge \square_{[0, 2]} q)$$

	0	1	2	3	4	5
p	True	True	True	True	False	False
q	True	True	True	True	True	True

# MLTL Rewrite Rules

$$\square_{[0, 3]} p \wedge \square_{[0, 5]} q$$

≡

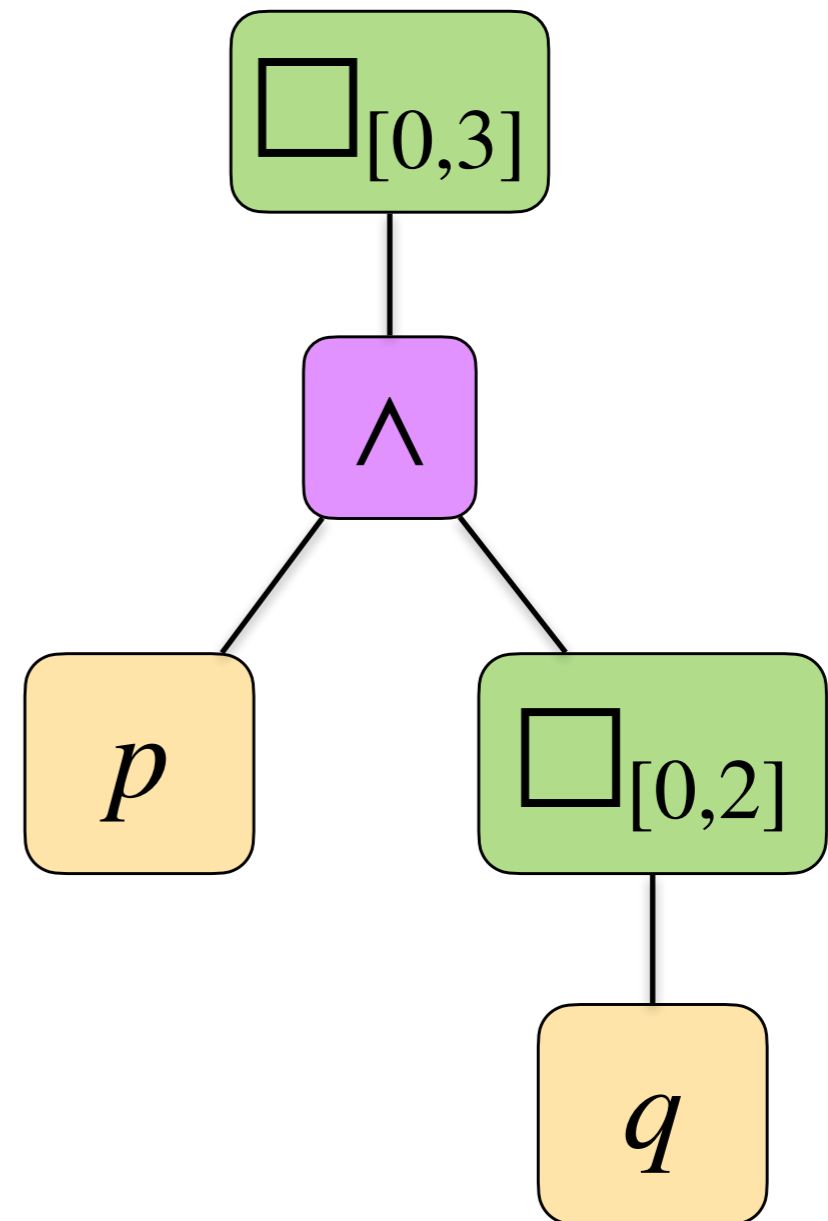
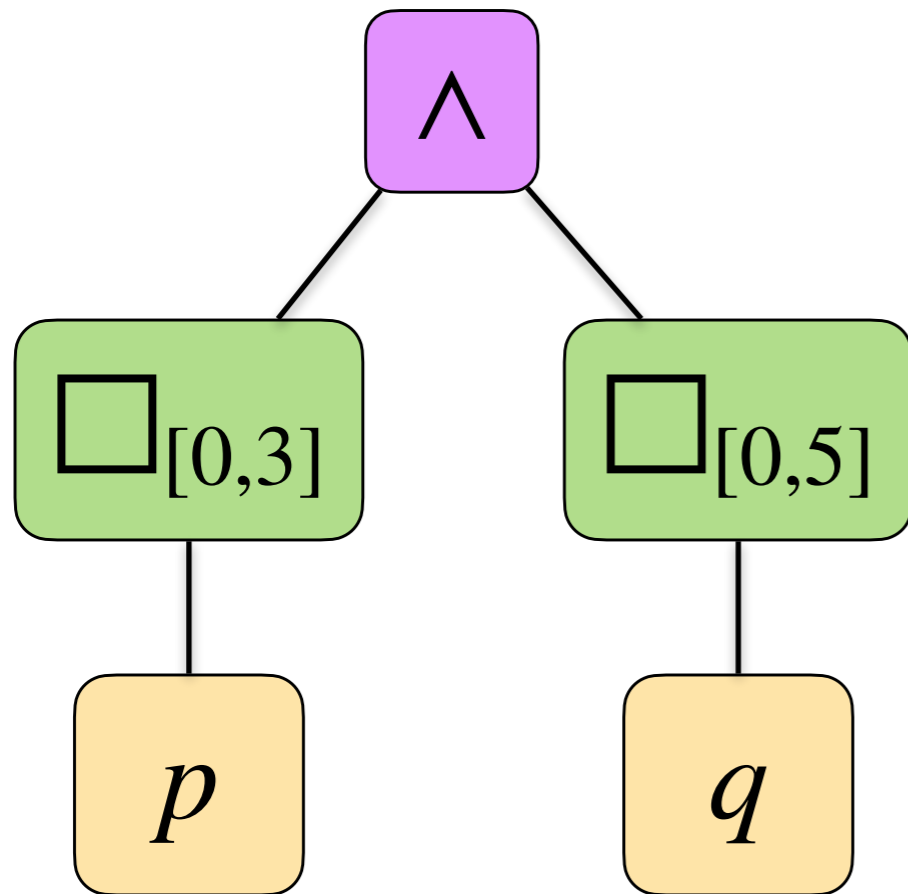
$$\square_{[0, 3]} (p \wedge \square_{[0, 2]} q)$$

	0	1	2	3	4	5
p						
q						

	0	1	2	3	4	5
p						
q						

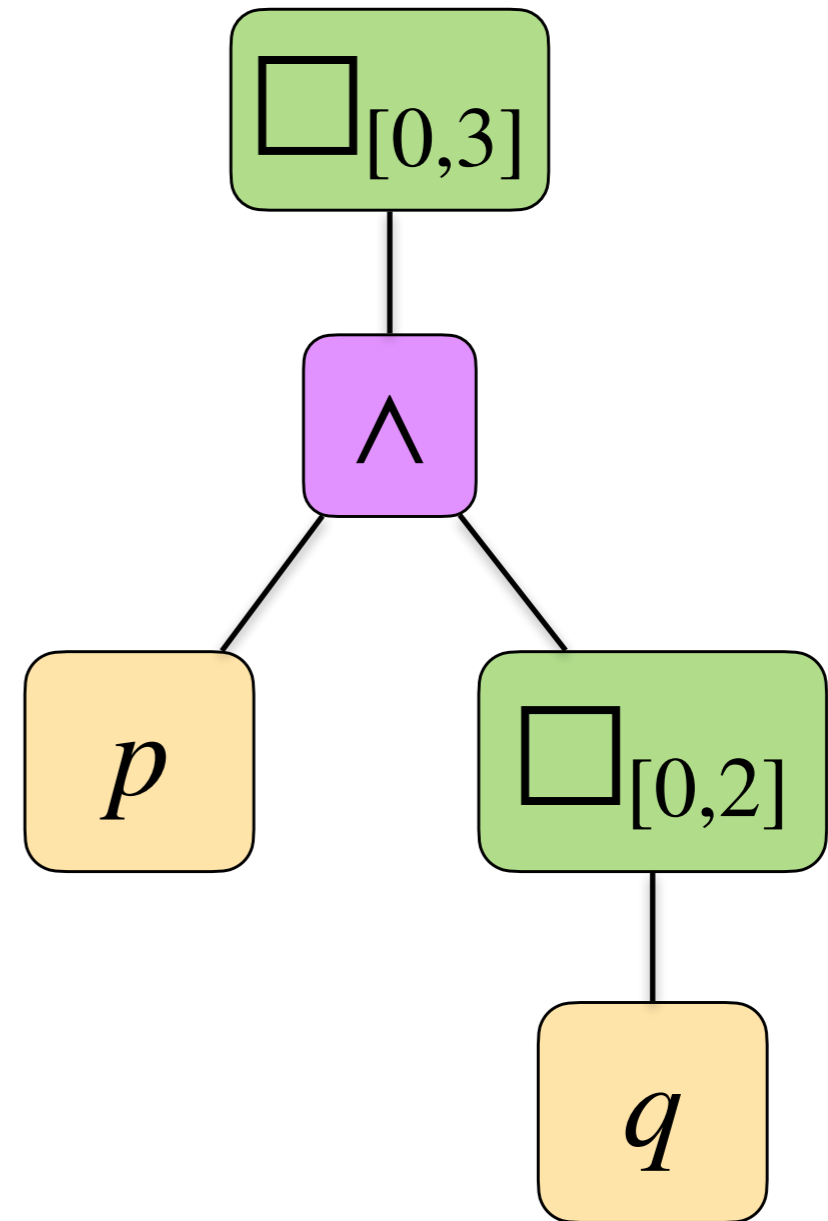
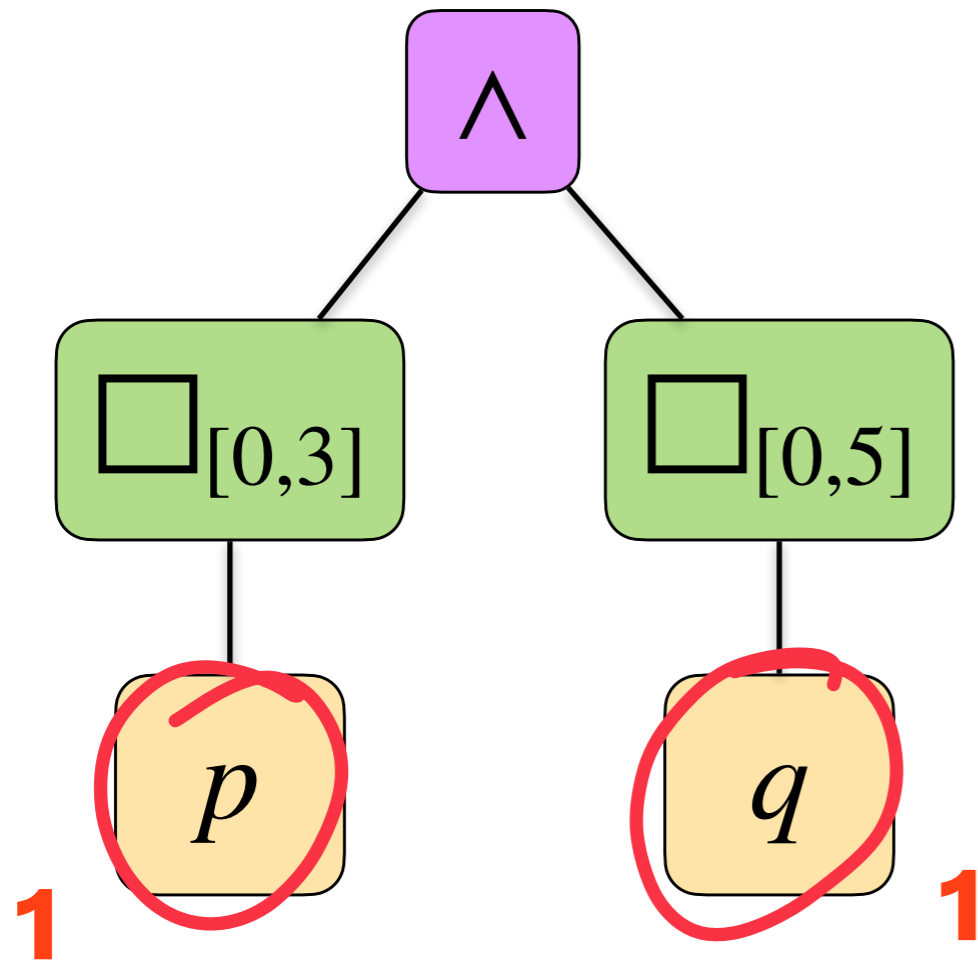
## MLTL Rewrite Rules

$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



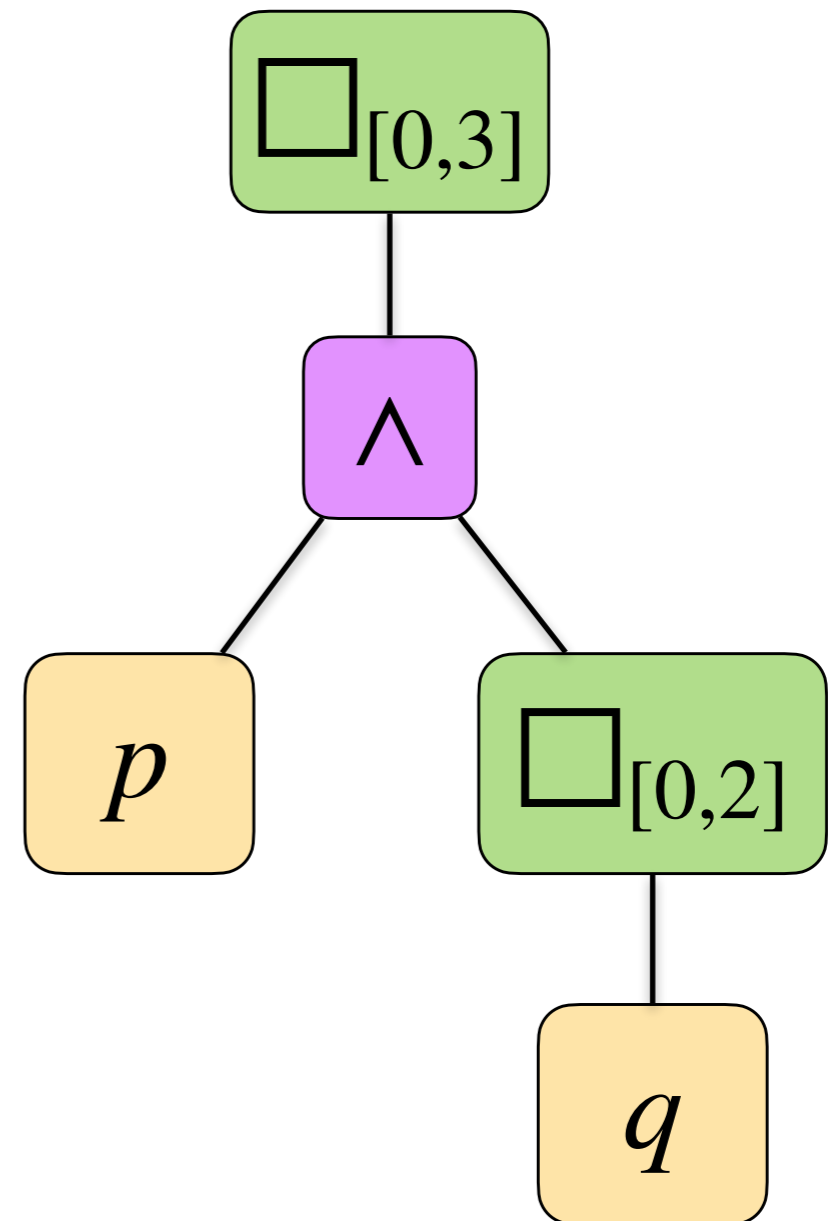
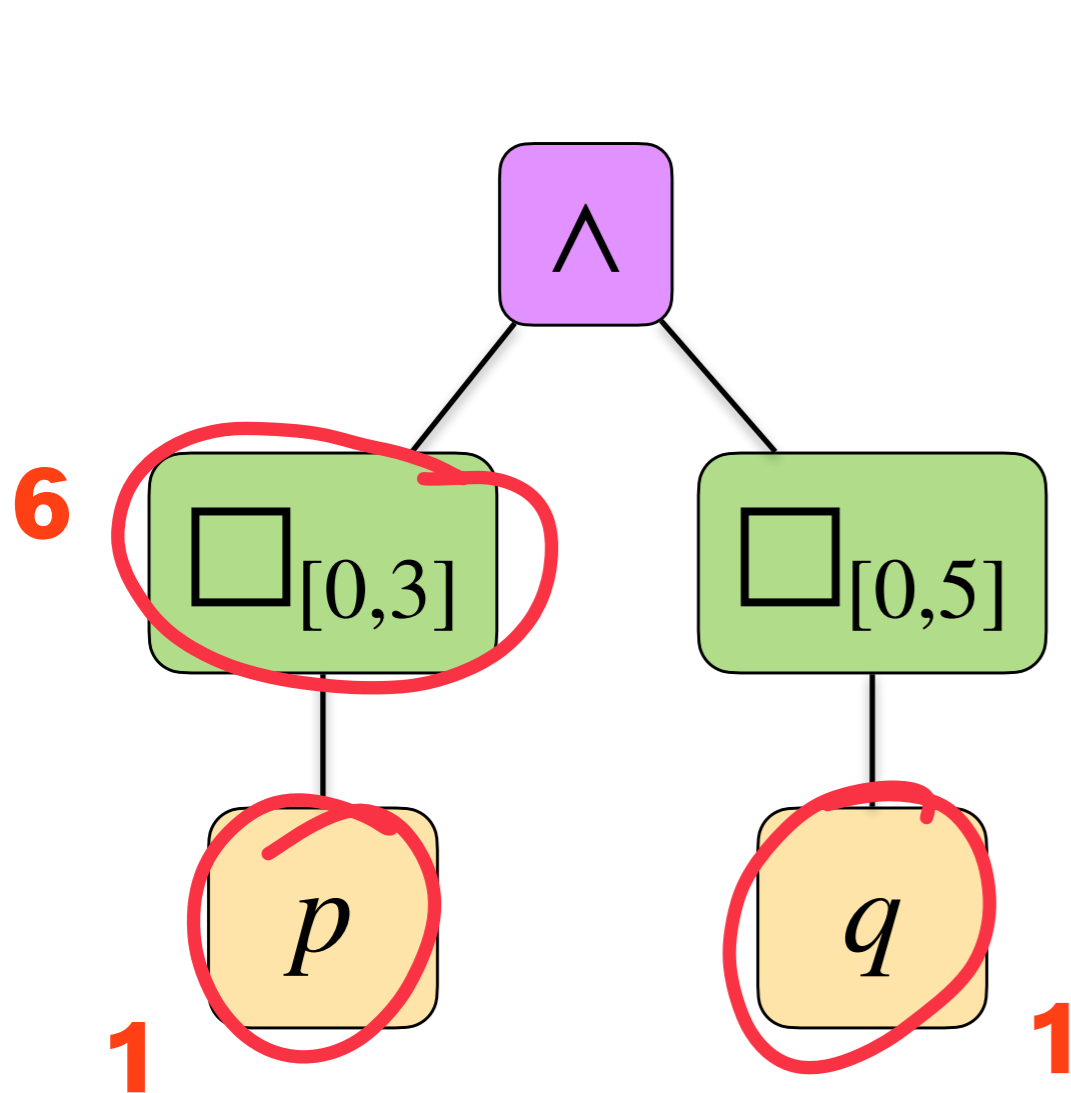
# MLTL Rewrite Rules

$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



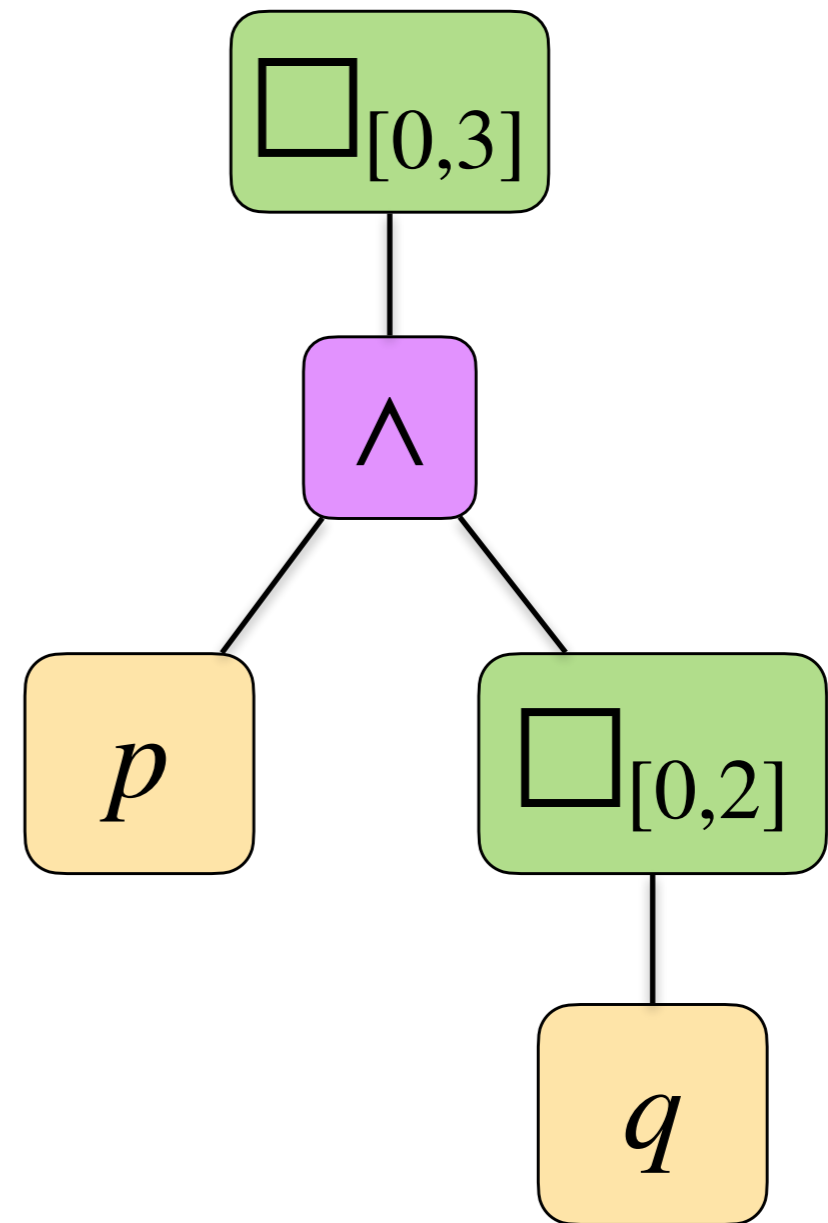
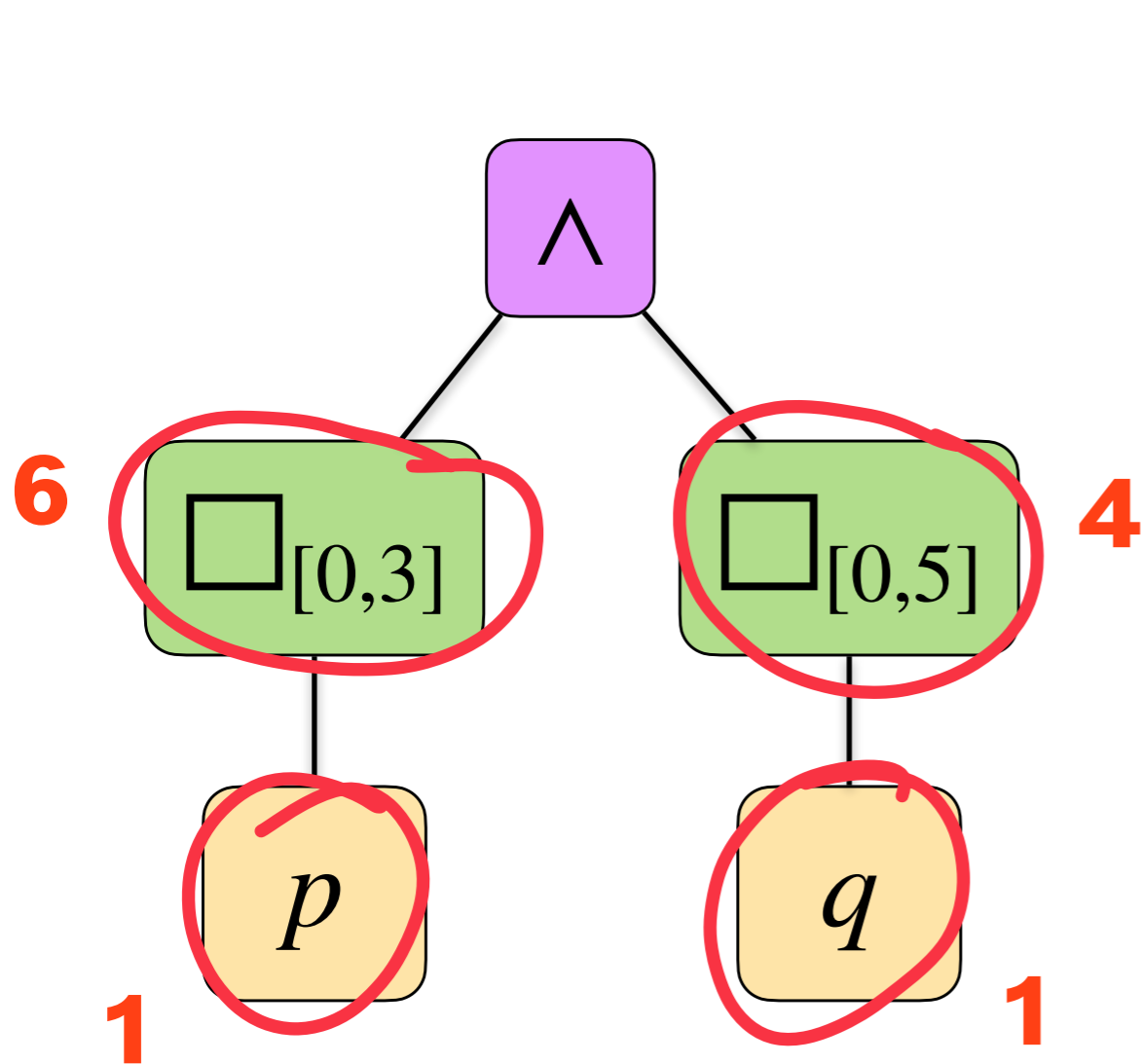
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$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



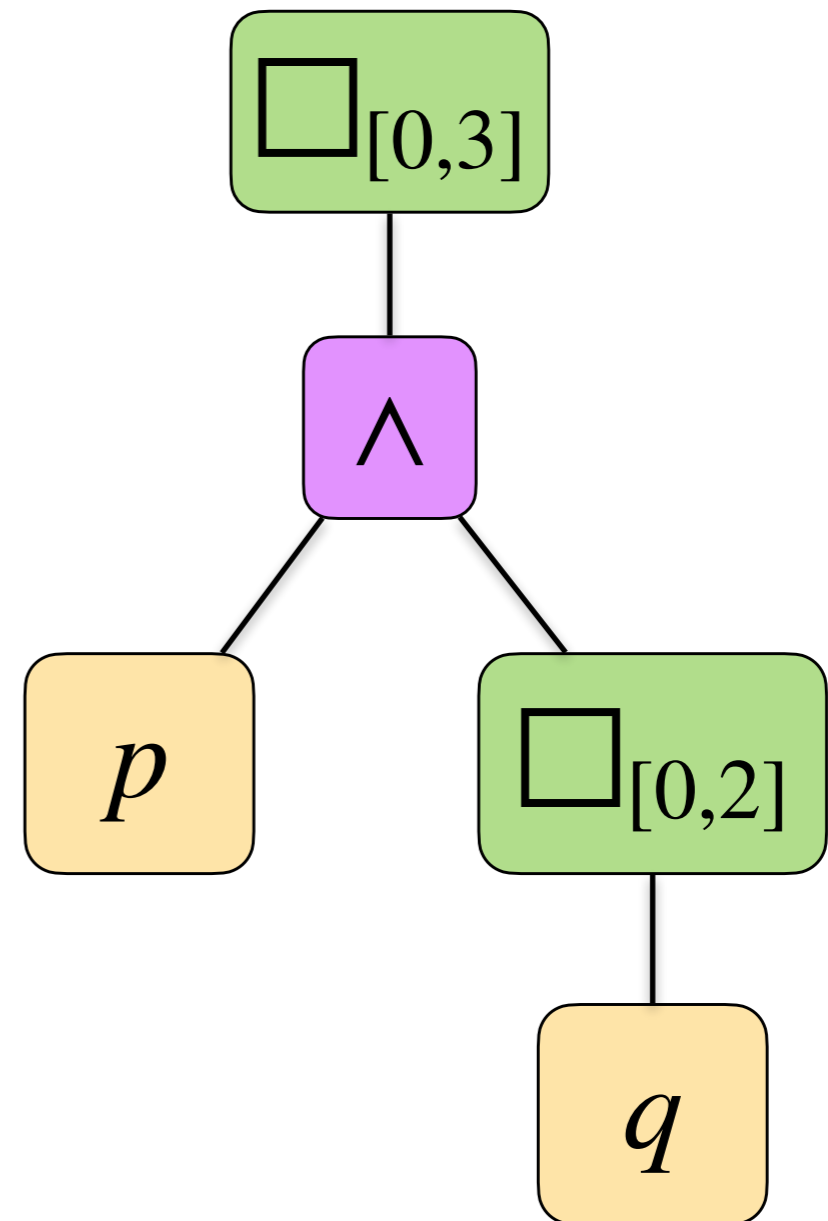
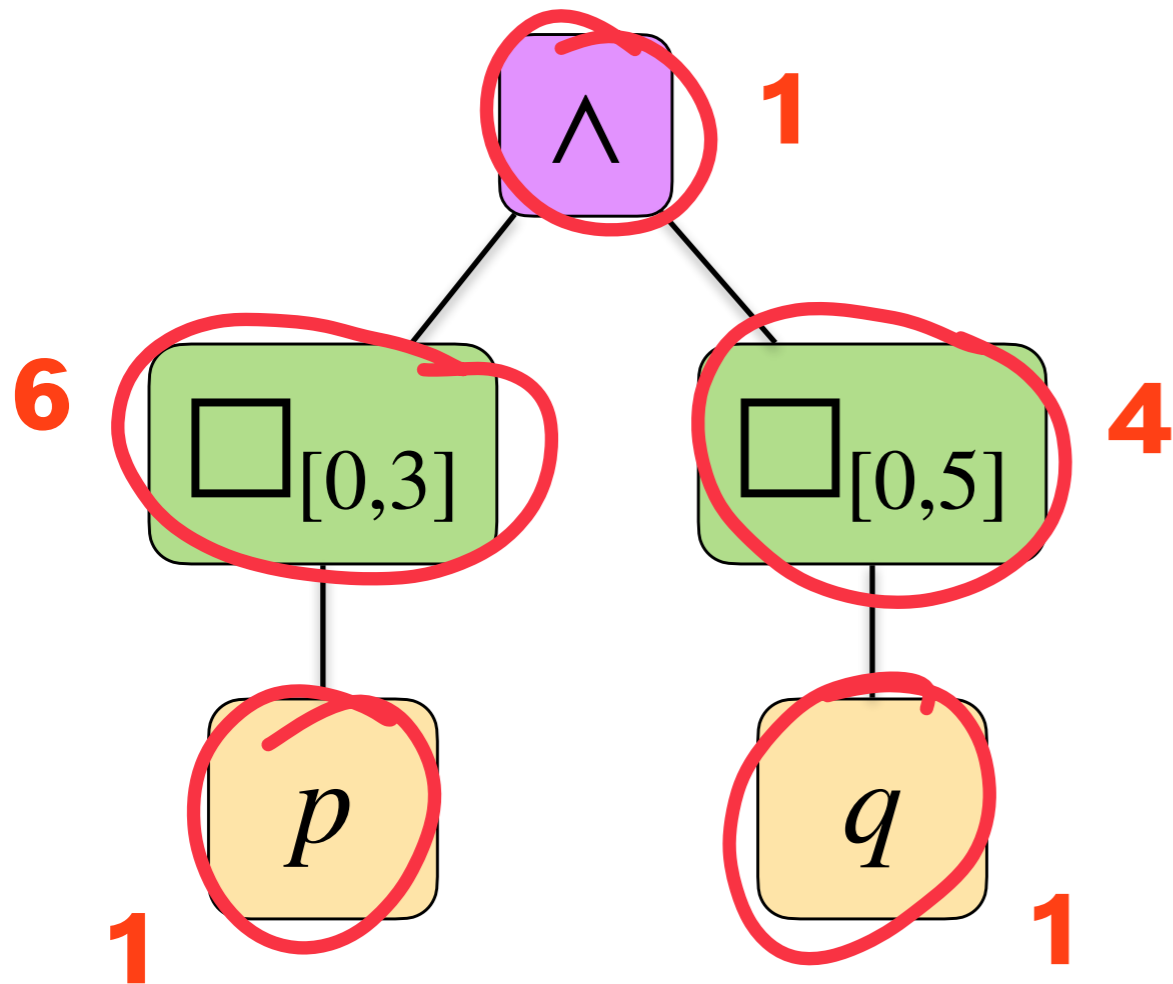
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$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



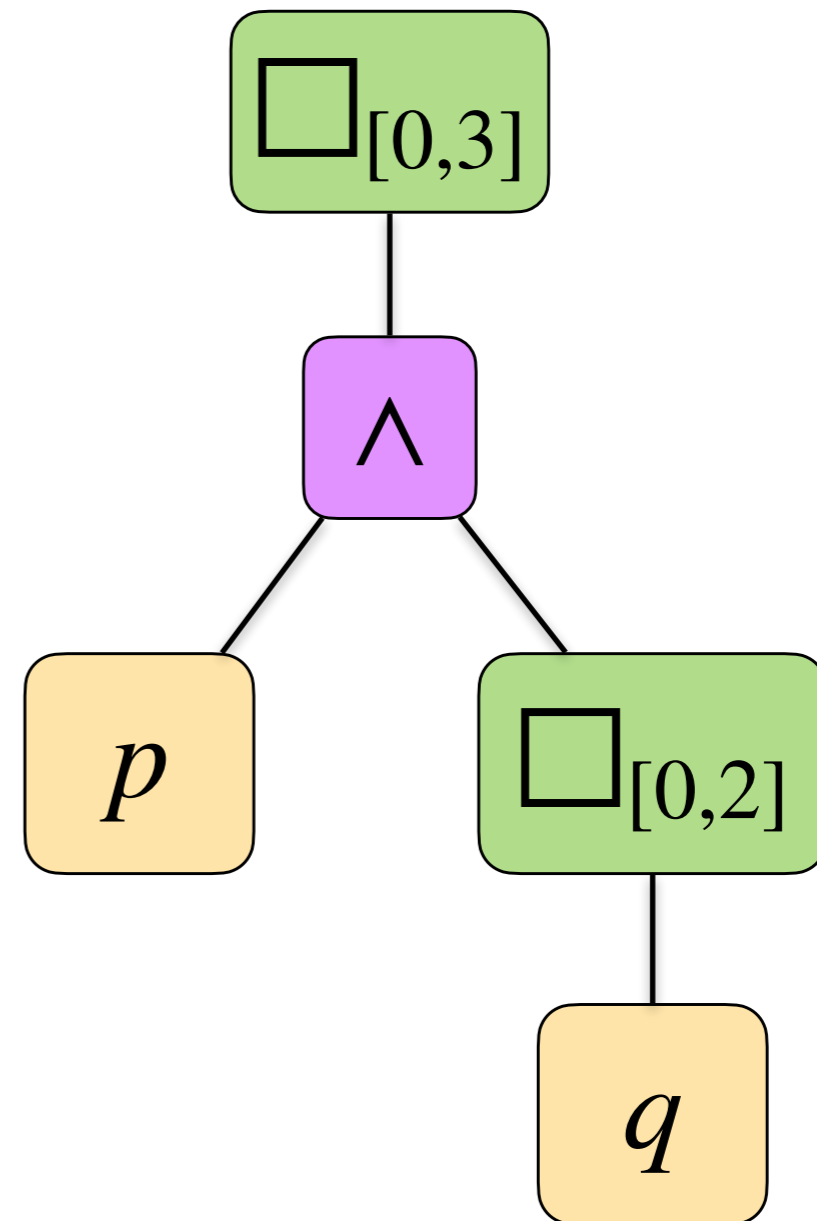
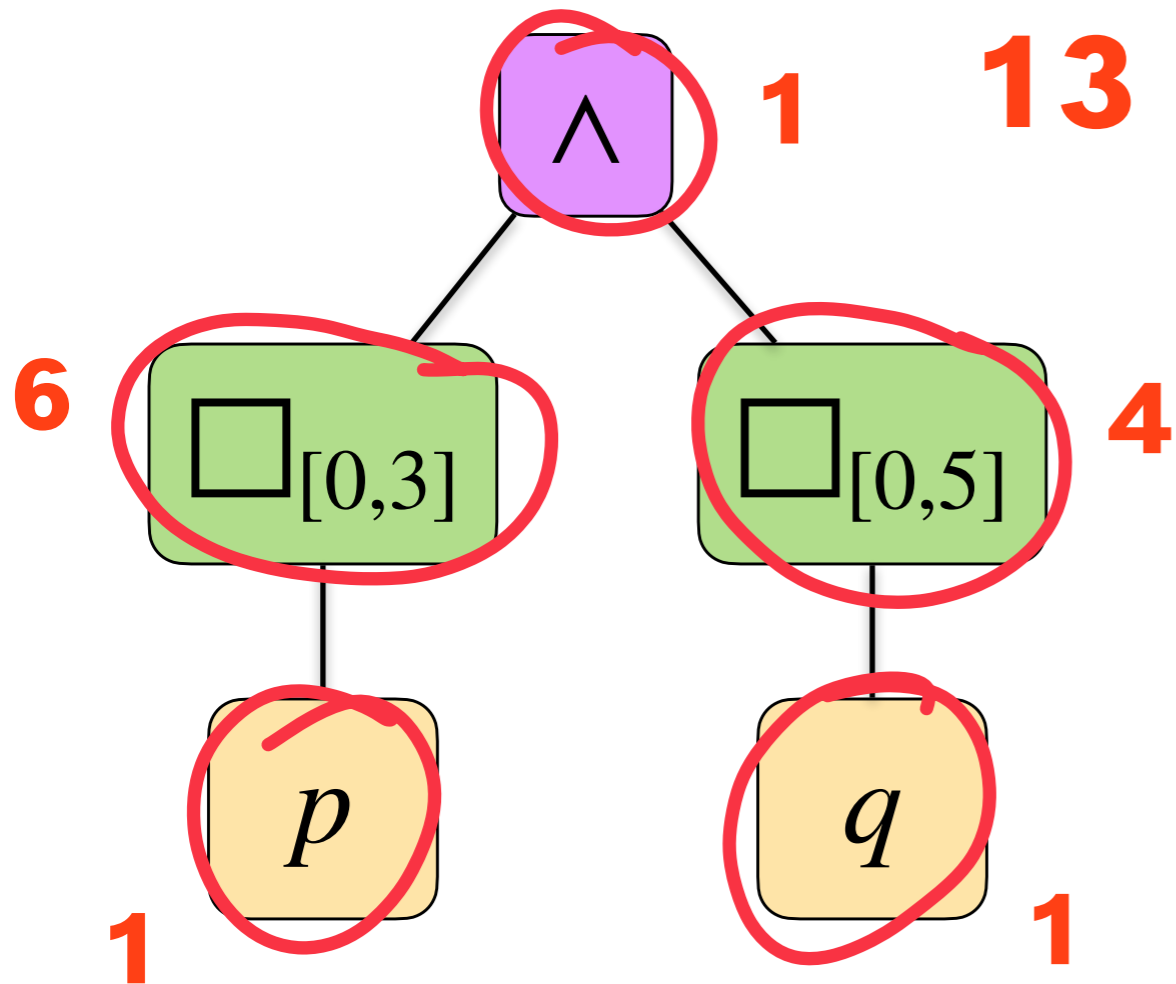
# MLTL Rewrite Rules

$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



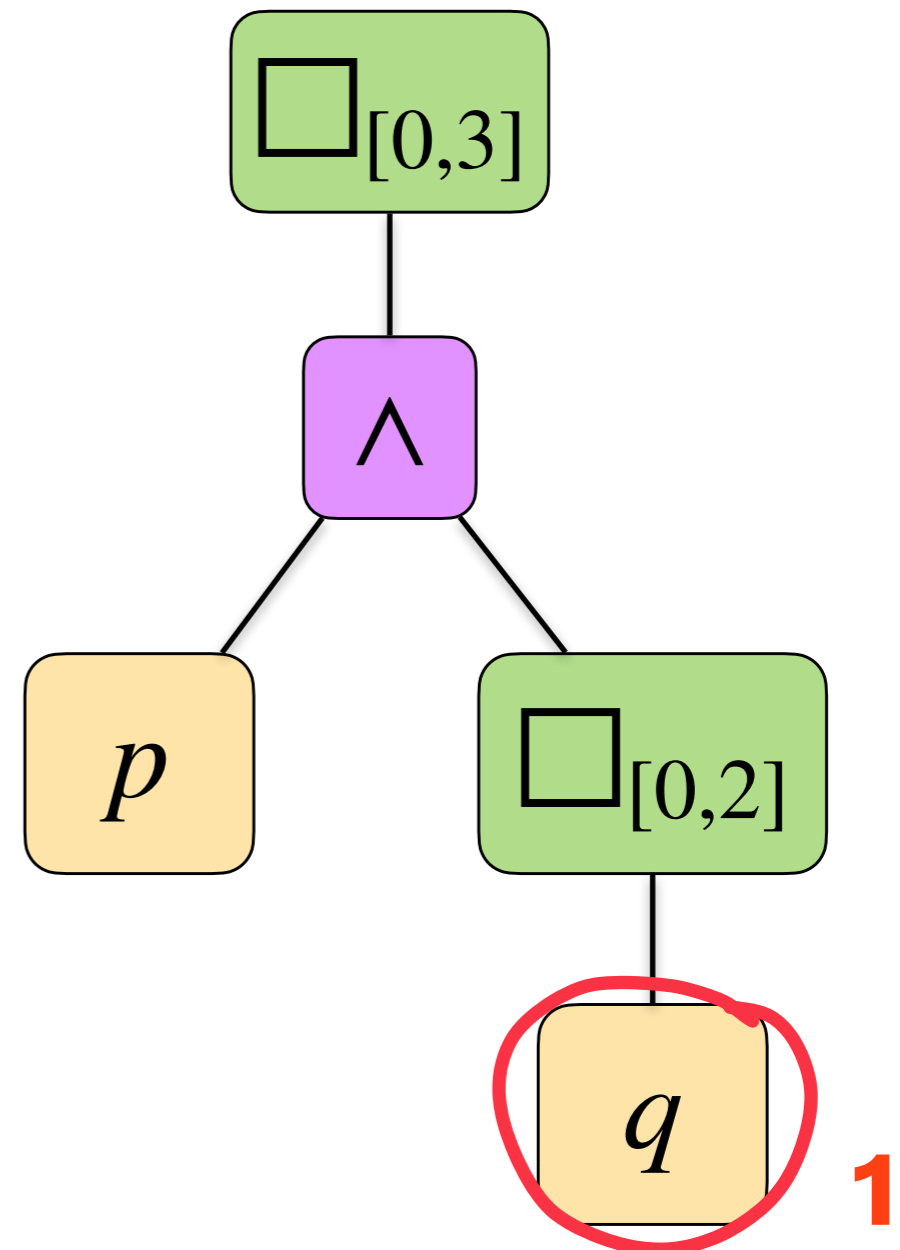
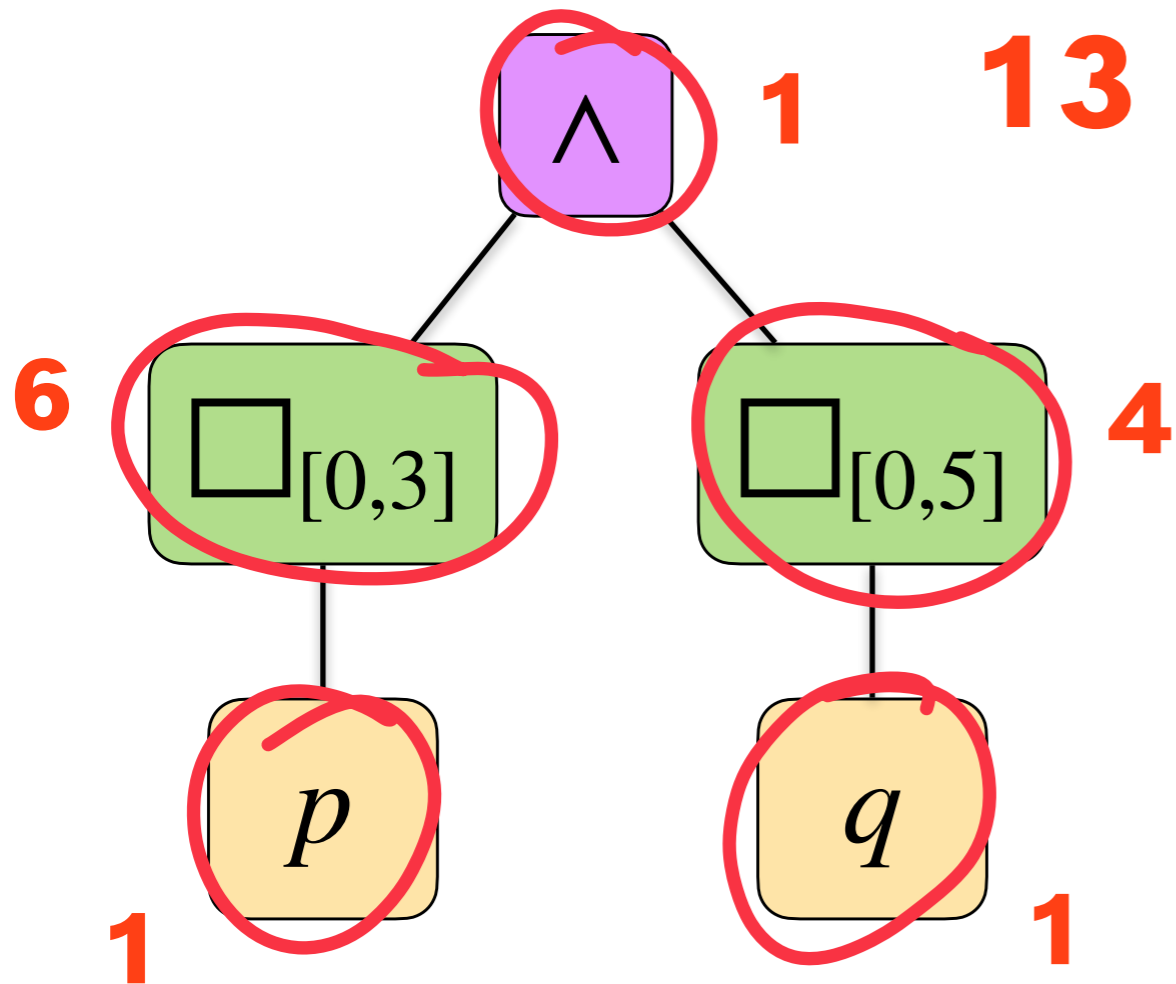
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$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



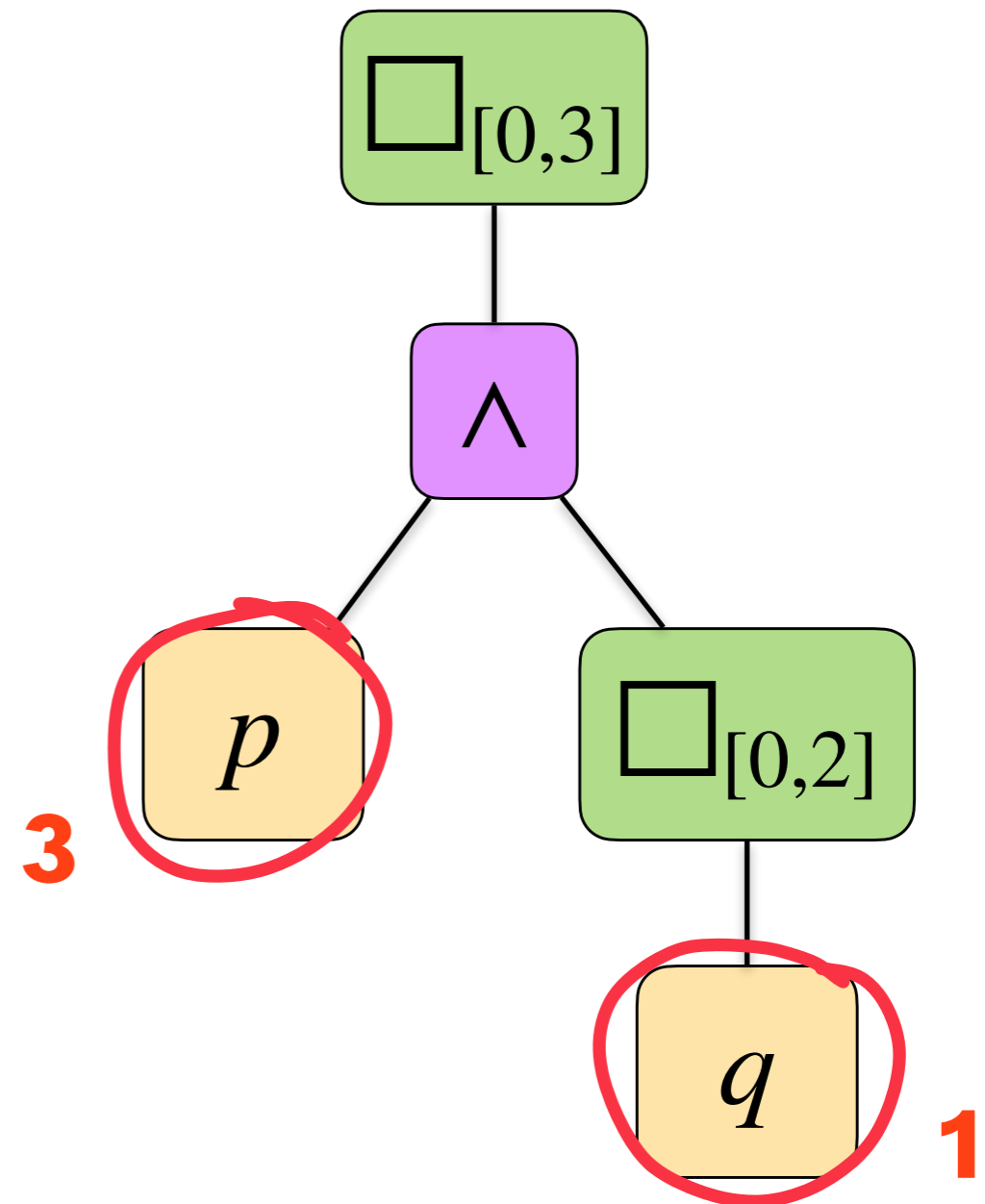
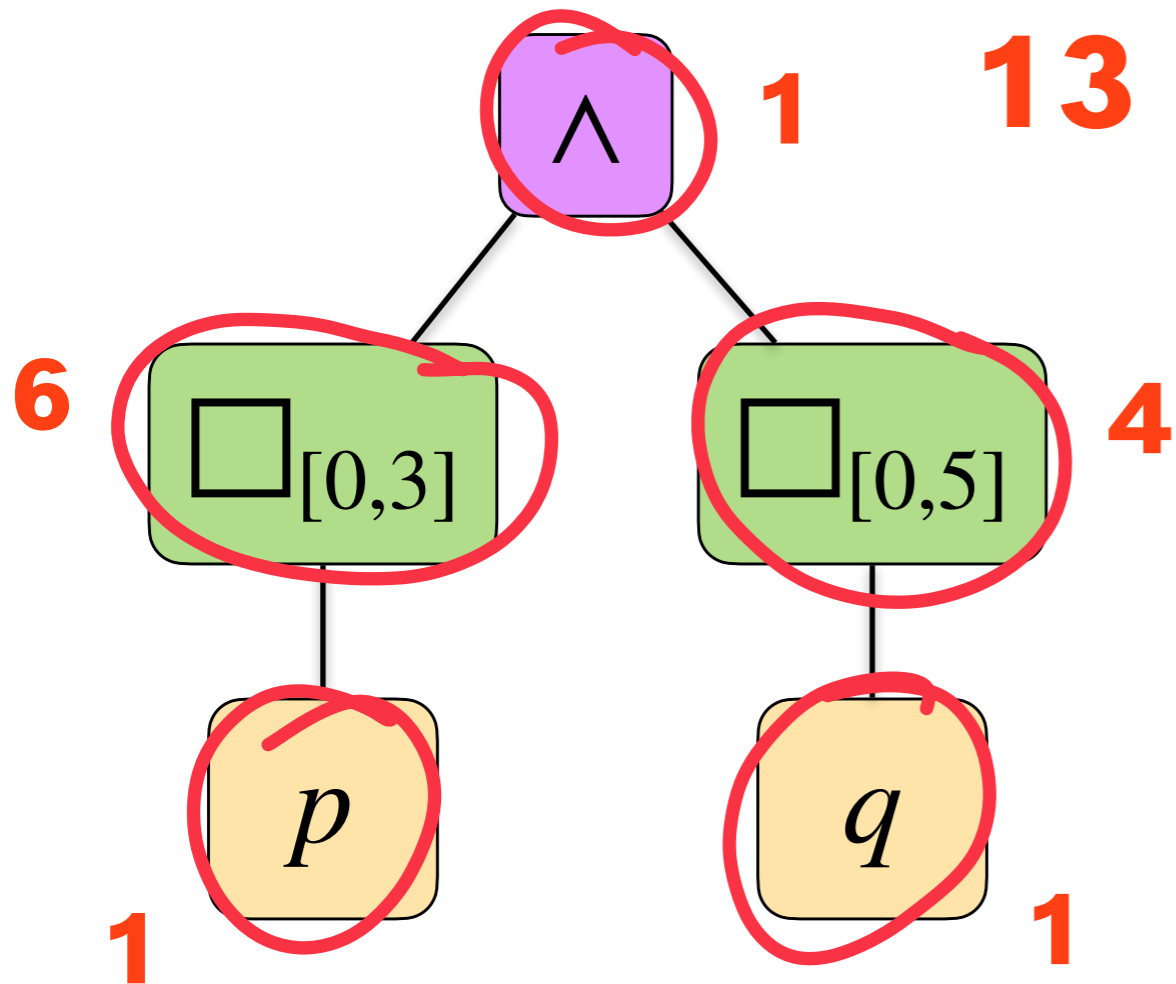
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$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



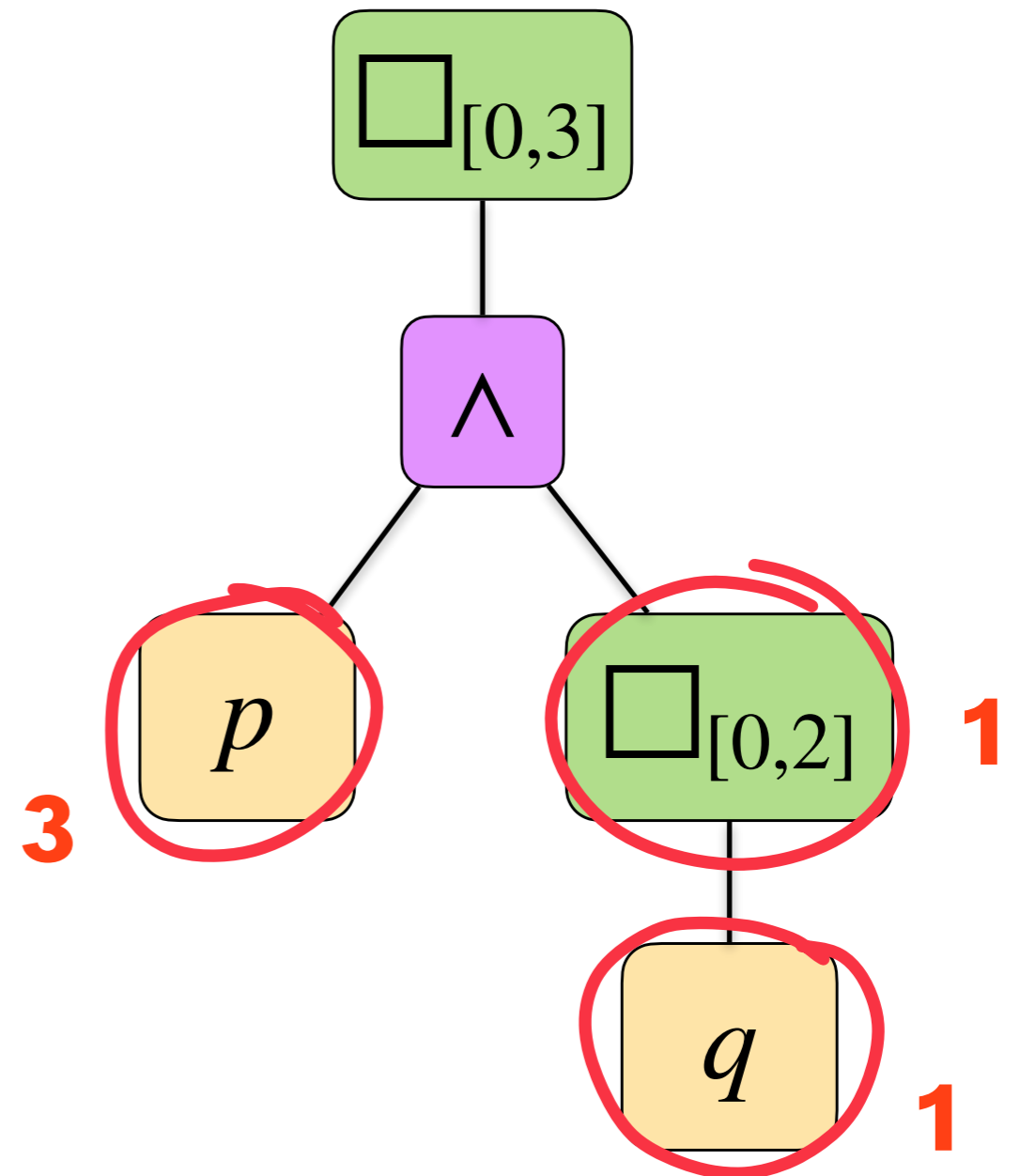
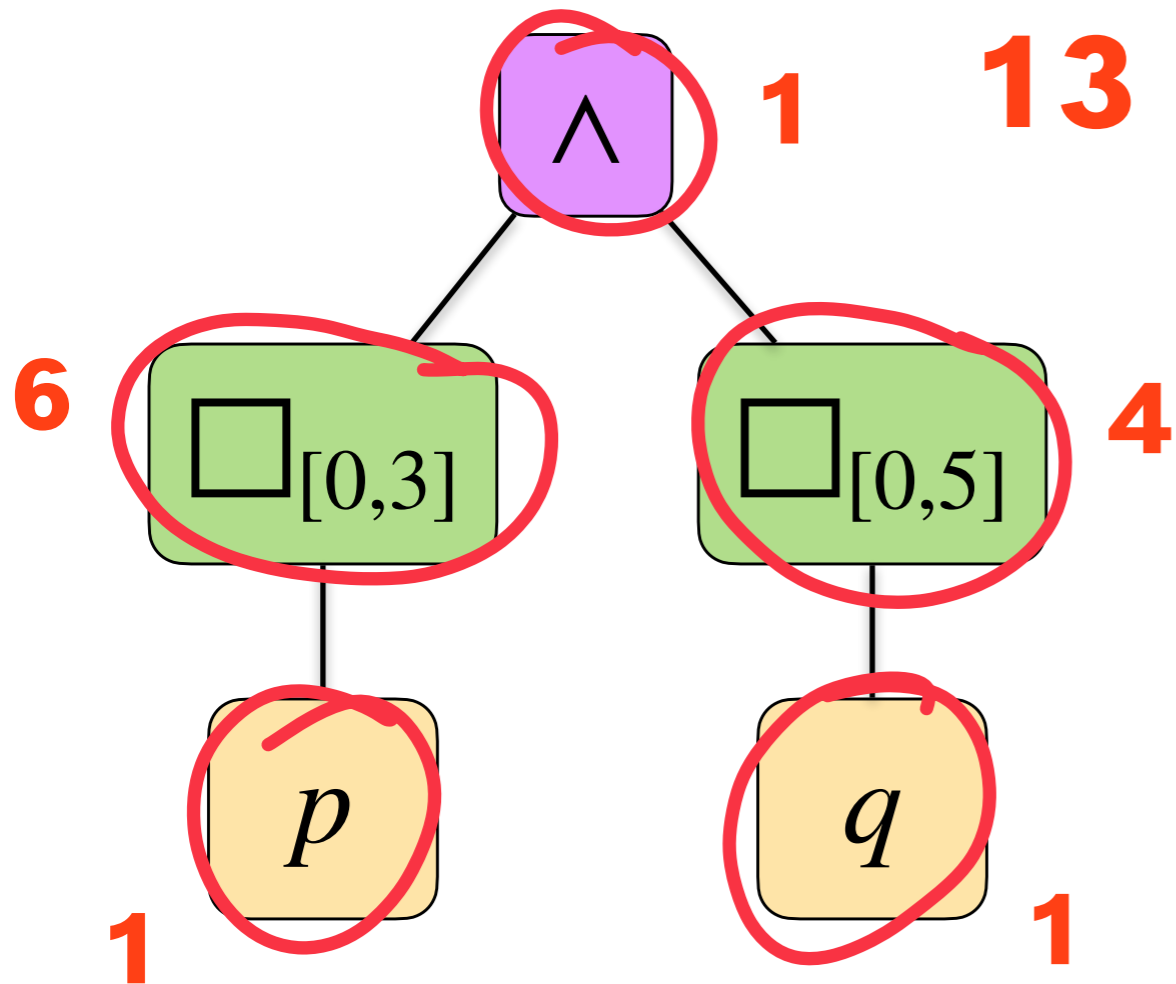
# MLTL Rewrite Rules

$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



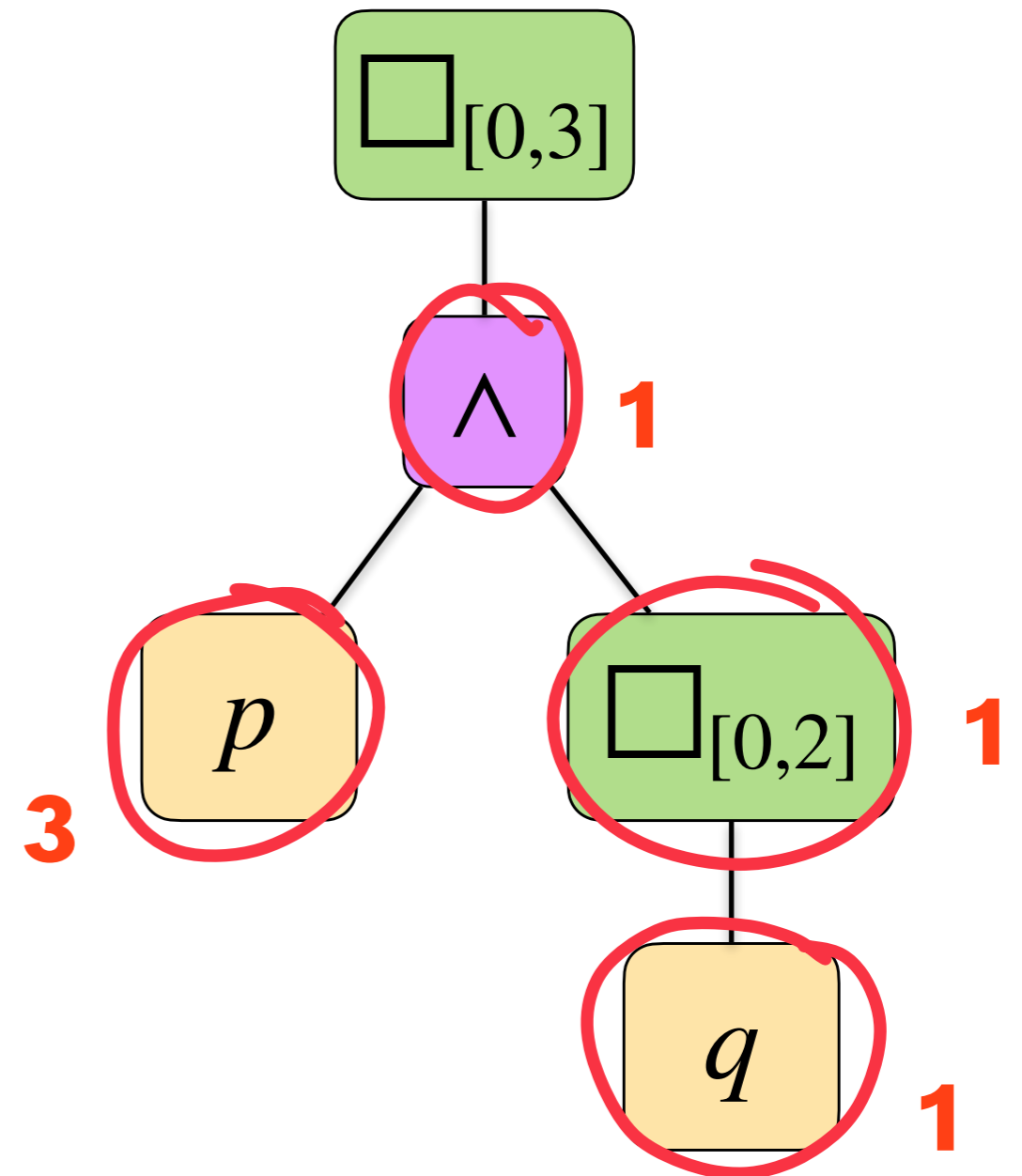
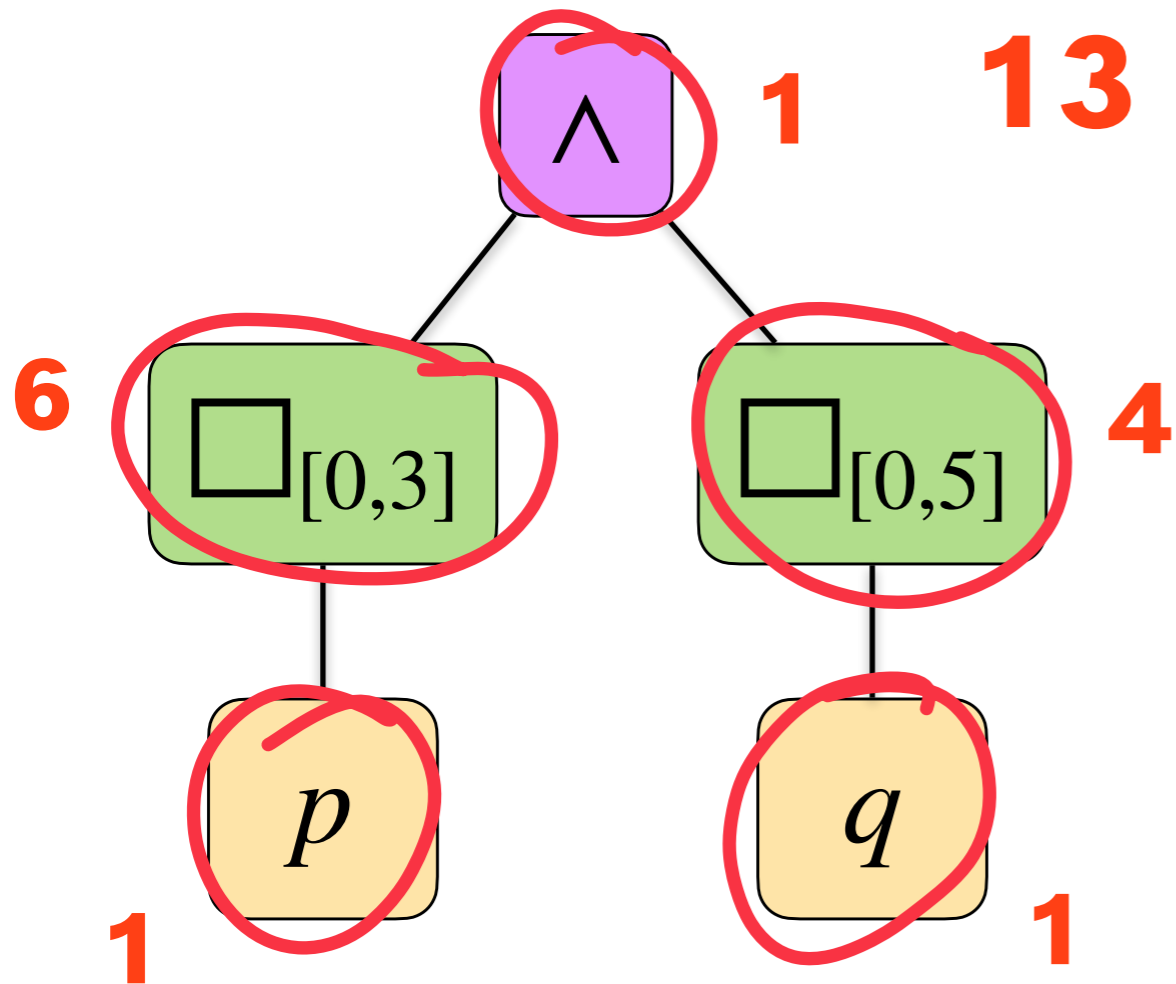
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$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



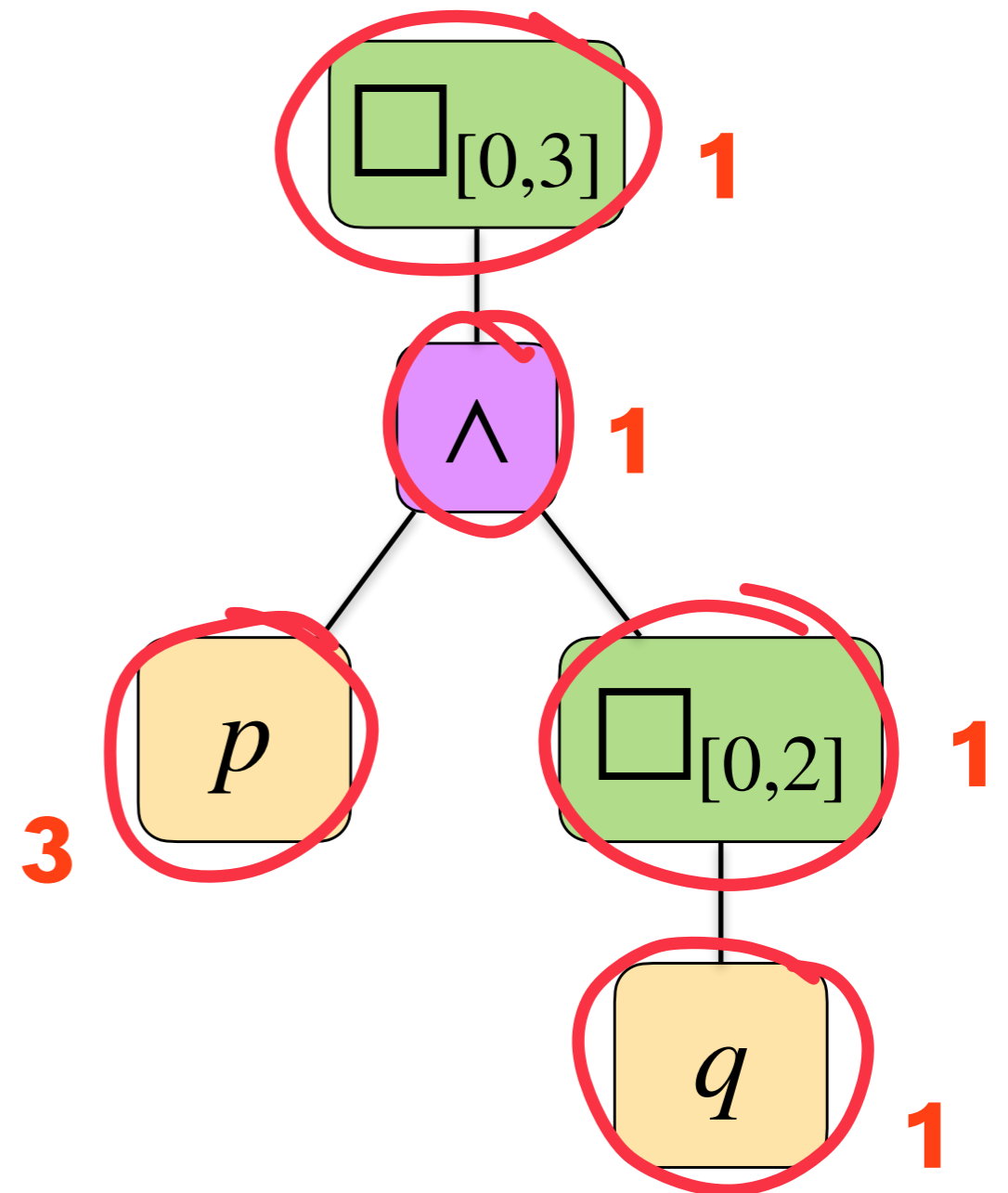
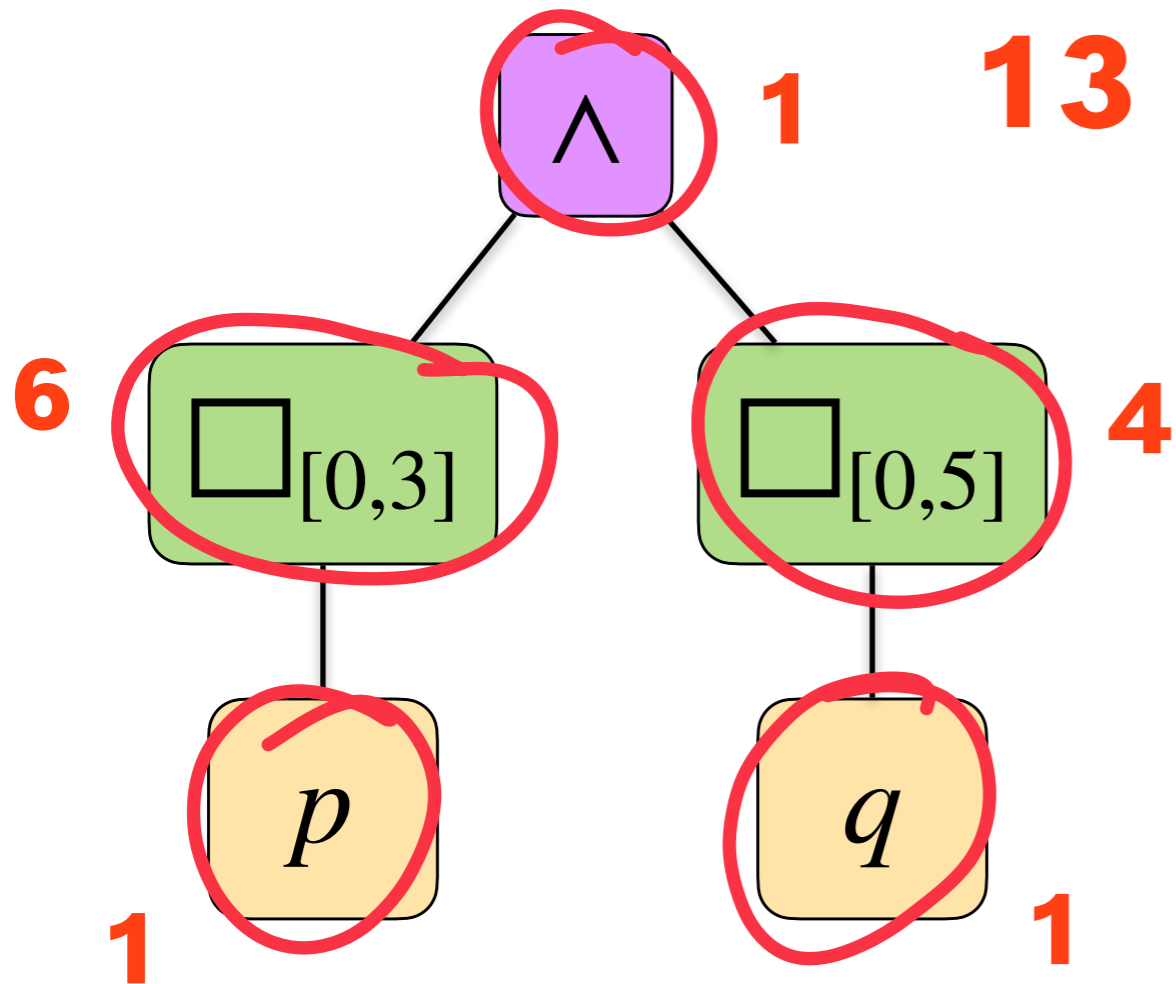
# MLTL Rewrite Rules

$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



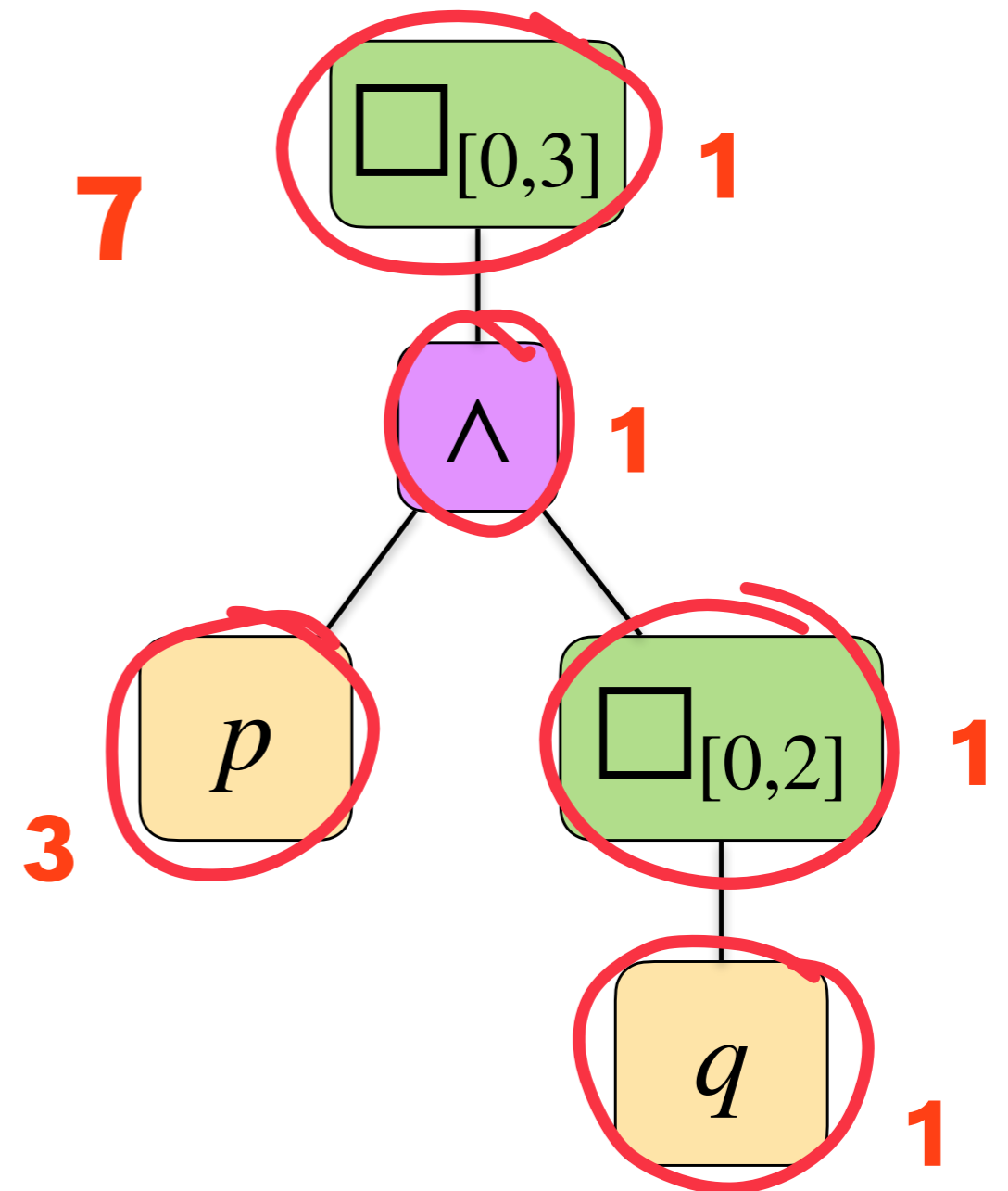
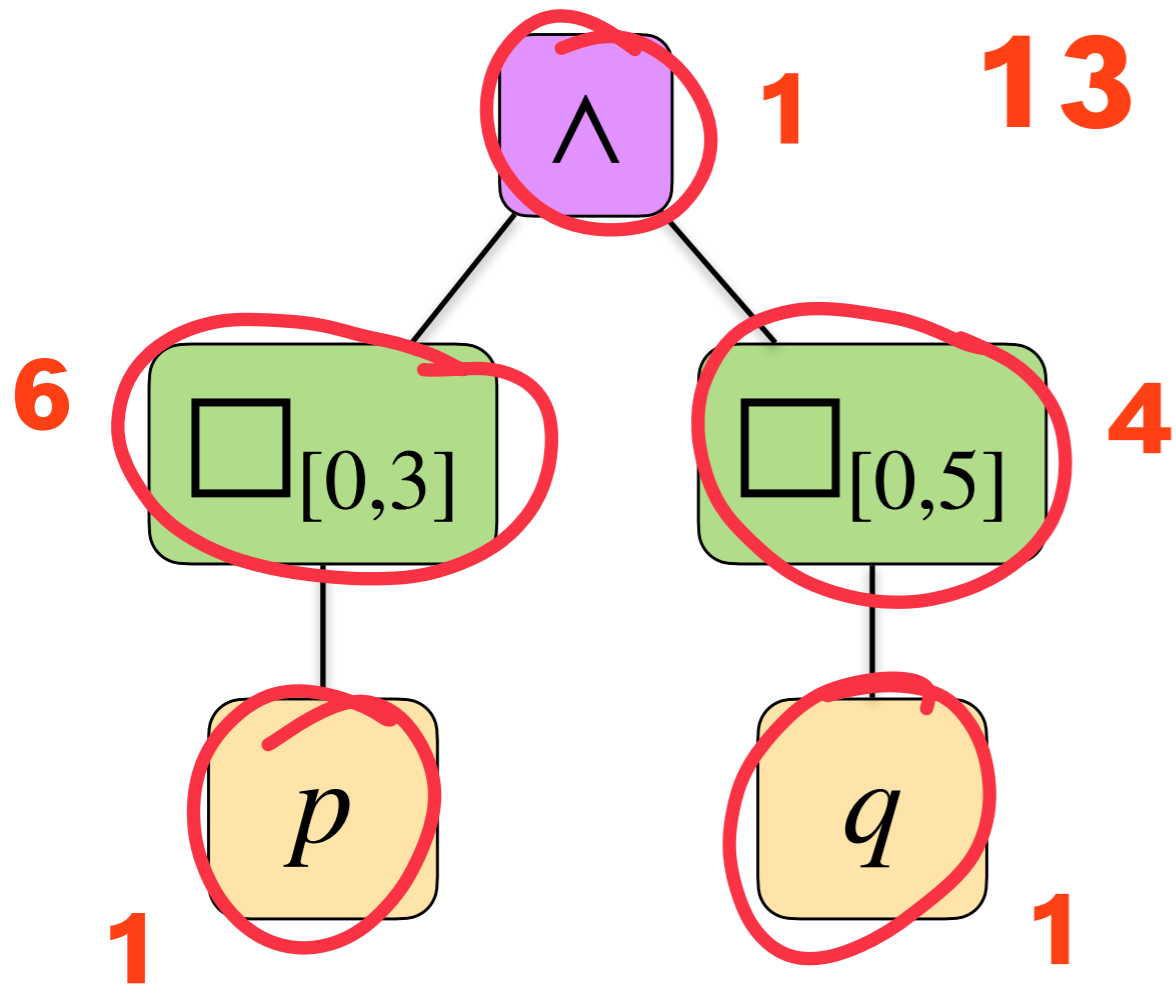
# MLTL Rewrite Rules

$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



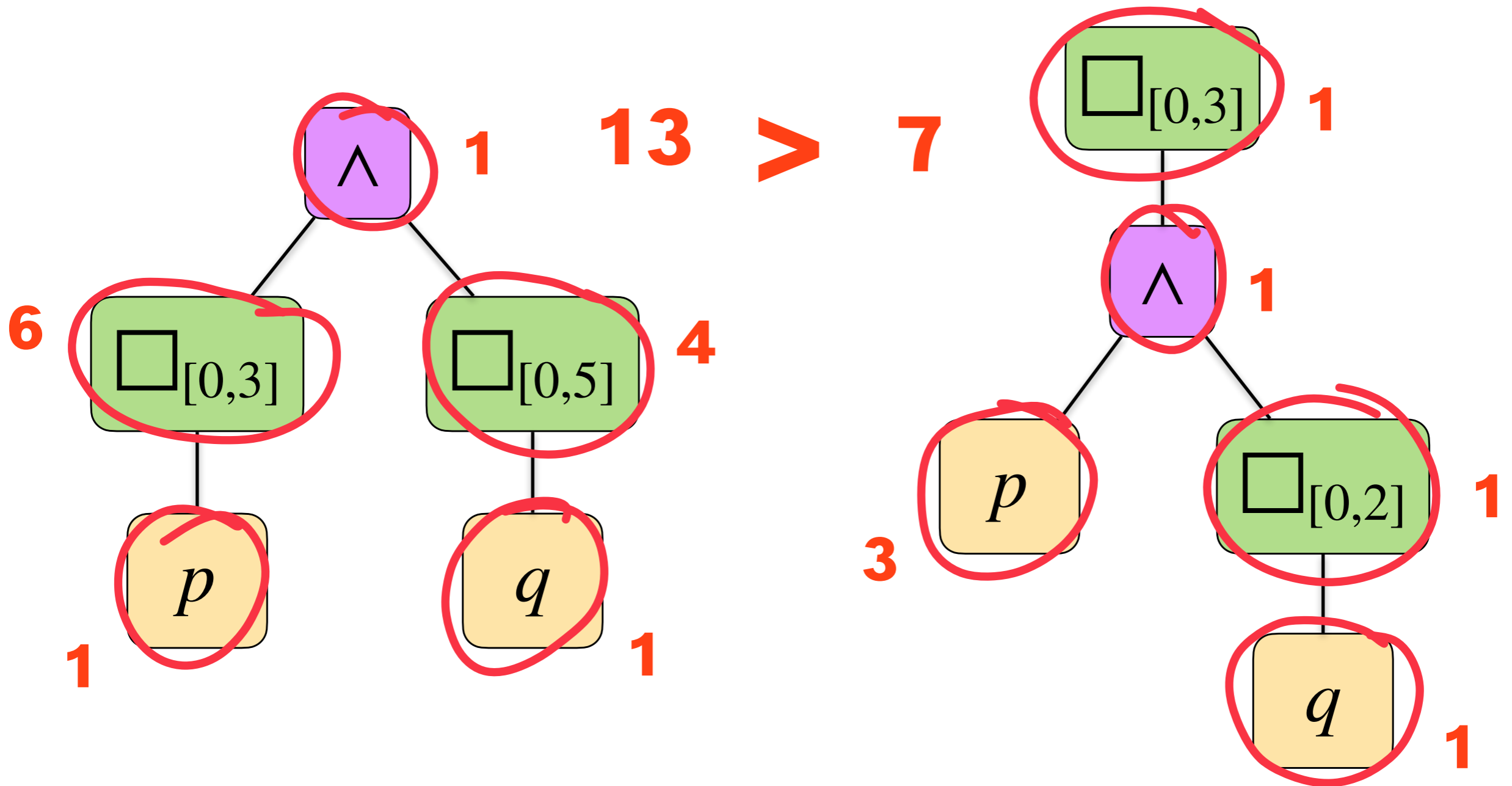
# MLTL Rewrite Rules

$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



# MLTL Rewrite Rules

$$\square_{[0,3]}p \wedge \square_{[0,5]}q \Rightarrow \square_{[0,3]}(p \wedge \square_{[0,2]}q)$$



## MLTL Rewrite Rules [19]

$\begin{array}{l} \Box_{[l_1, u_1]} \Box_{[l_2, u_2]} \varphi \mapsto \\ \Box_{[l_1 + l_2, u_1 + u_2]} \varphi \end{array}$	$\begin{array}{l} \Diamond_{[l_1, u_1]} \Diamond_{[l_2, u_2]} \varphi \mapsto \\ \Diamond_{[l_1 + l_2, u_1 + u_2]} \varphi \end{array}$	(R1)
<div style="border: 2px solid red; padding: 5px; margin: 5px 0;"> <math display="block">\Box_{[l_1, u_1]} \varphi \wedge \Box_{[l_2, u_2]} \psi \mapsto \Box_{[l_3, u_3]} (\Box_{[l_1 - l_3, u_1 - u_3]} \varphi \wedge \Box_{[l_2 - l_3, u_2 - u_3]} \psi)</math> </div>		
$\Diamond_{[l_1, u_1]} \varphi \vee \Diamond_{[l_2, u_2]} \psi \mapsto \Diamond_{[l_3, u_3]} (\Diamond_{[l_1 - l_3, u_1 - u_3]} \varphi \vee \Diamond_{[l_2 - l_3, u_2 - u_3]} \psi)$		(R2)
<p style="text-align: center;">where <math>l_3 = \min(l_1, l_2)</math>, <math>u_3 = l_3 + \min(u_1 - l_1, u_2 - l_2)</math>, <math>l_3 &lt; u_3</math></p>		
$\Box_{[a, a]} \Diamond_{[l, u]} \varphi \mapsto \Diamond_{[l + a, u + a]} \varphi$	$\Diamond_{[l, u]} \Box_{[a, a]} \varphi \mapsto \Diamond_{[l + a, u + a]} \varphi$	(R3)
$\Diamond_{[a, a]} \Box_{[l, u]} \varphi \mapsto \Box_{[l + a, u + a]} \varphi$	$\Box_{[l, u]} \Diamond_{[a, a]} \varphi \mapsto \Box_{[l + a, u + a]} \varphi$	
$\Box_{[l_1, u_1]} \varphi \wedge \Box_{[l_2, u_2]} \varphi \mapsto \Box_{[l_1, u_3]} \varphi \quad \Diamond_{[l_1, u_1]} \varphi \vee \Diamond_{[l_2, u_2]} \varphi \mapsto \Diamond_{[l_1, u_2]} \varphi$		(R4)
<p style="text-align: center;">where <math>l_1 \leq l_2 \leq u_1 + 1</math>, <math>u_3 = \max(u_1, u_2)</math></p>		
$\Box_{[l_1, u_1]} \varphi \vee \Box_{[l_2, u_2]} \varphi \mapsto \Box_{[l_2, u_2]} \varphi \quad \Diamond_{[l_1, u_1]} \varphi \wedge \Diamond_{[l_2, u_2]} \varphi \mapsto \Diamond_{[l_2, u_2]} \varphi$		(R5)
<p style="text-align: center;">where <math>l_1 \leq l_2 \leq u_2 \leq u_1</math></p>		
$\Box_{[a, a]} (\varphi \mathcal{U}_{[l, u]} \psi) \mapsto$	$(\Box_{[a, a]} \varphi) \mathcal{U}_{[l, u]} (\Box_{[a, a]} \psi) \mapsto$	(R6)
$\varphi \mathcal{U}_{[l + a, u + a]} \psi$	$\varphi \mathcal{U}_{[l + a, u + a]} \psi$	
$(\varphi_1 \mathcal{U}_{[l, u_1]} \varphi_2) \wedge (\varphi_3 \mathcal{U}_{[l, u_2]} \varphi_2) \mapsto (\varphi_1 \wedge \varphi_3) \mathcal{U}_{[l, u_1]} \varphi_2$		(R7)
<p style="text-align: center;">where <math>l \leq u_1, l \leq u_2, u_1 \leq u_2</math></p>		
$\varphi \mathcal{U}_{[l_1, u_1]} \Box_{[0, u_2]} \varphi \mapsto \Box_{[l_1, l_1 + u_2]} \varphi$		(R8)
$\varphi \mathcal{U}_{[l_1, u_1]} \Diamond_{[0, u_2]} \varphi \mapsto \Diamond_{[l_1, l_1 + u_2]} \varphi$		

## How to apply rewrite rules algorithmically?

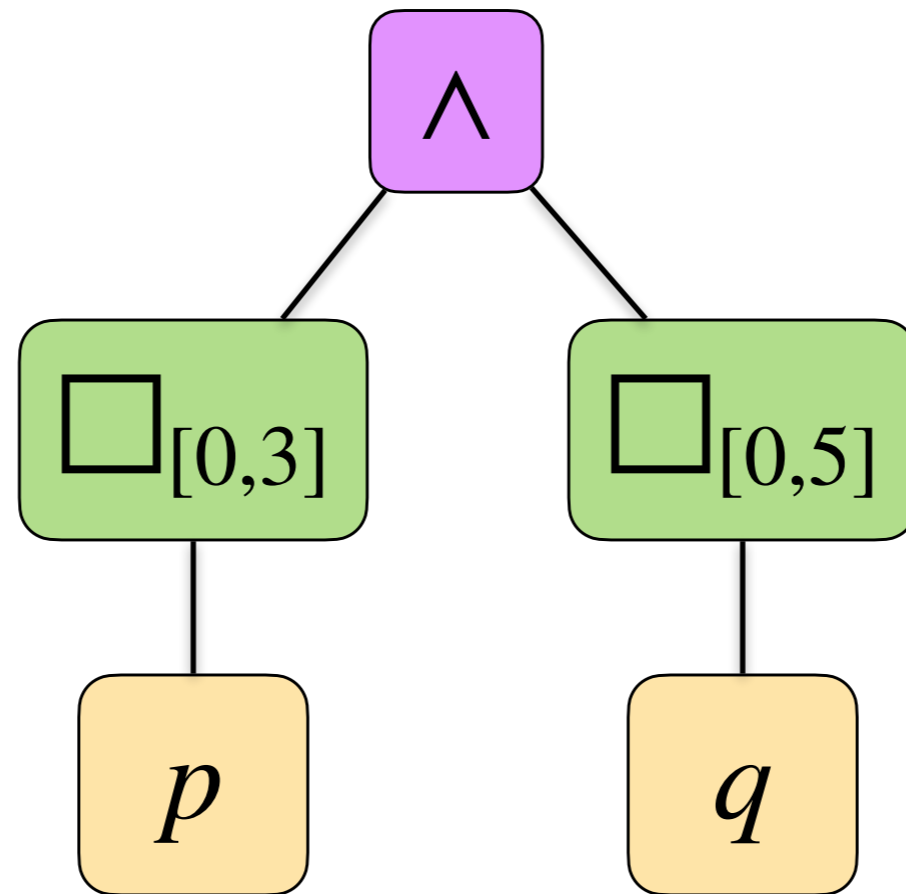
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- > **Order of rewrites matter** — applying one rule may **disable** the application of another.
  - > Assume  $R_1$  disables  $R_3$
  - >  $R_1 \longrightarrow R_2$
  - >  $R_2 \longrightarrow R_3 \longrightarrow R_1$
  - > ...
- > Classic problem in **program optimization [20]**
- > Solution: **Equality Saturation [21]**

# MLTL Equality Saturation

---

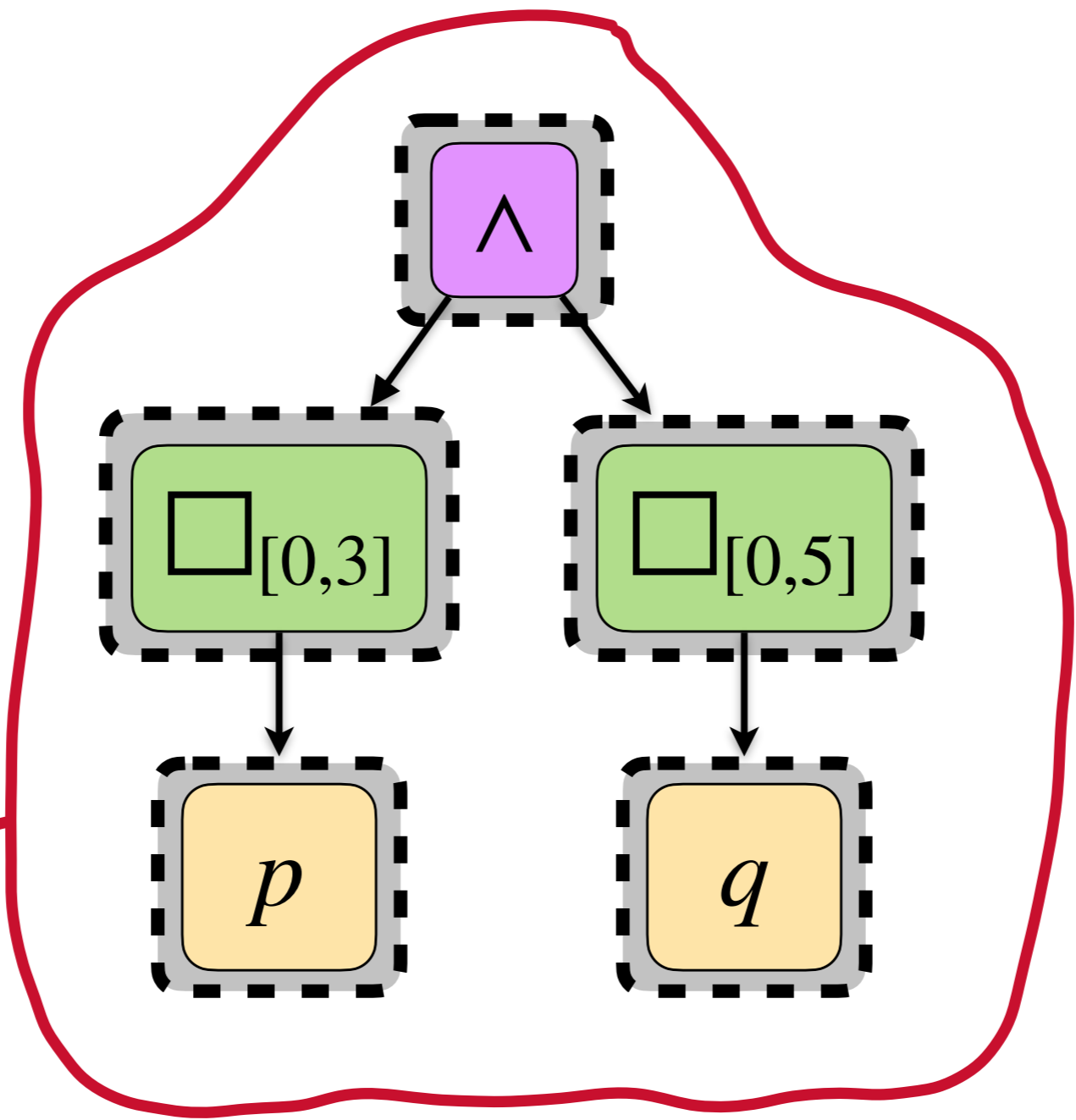
$$\Box_{[0,3]} p \wedge \Box_{[0,5]} q$$



# MLTL Equality Saturation

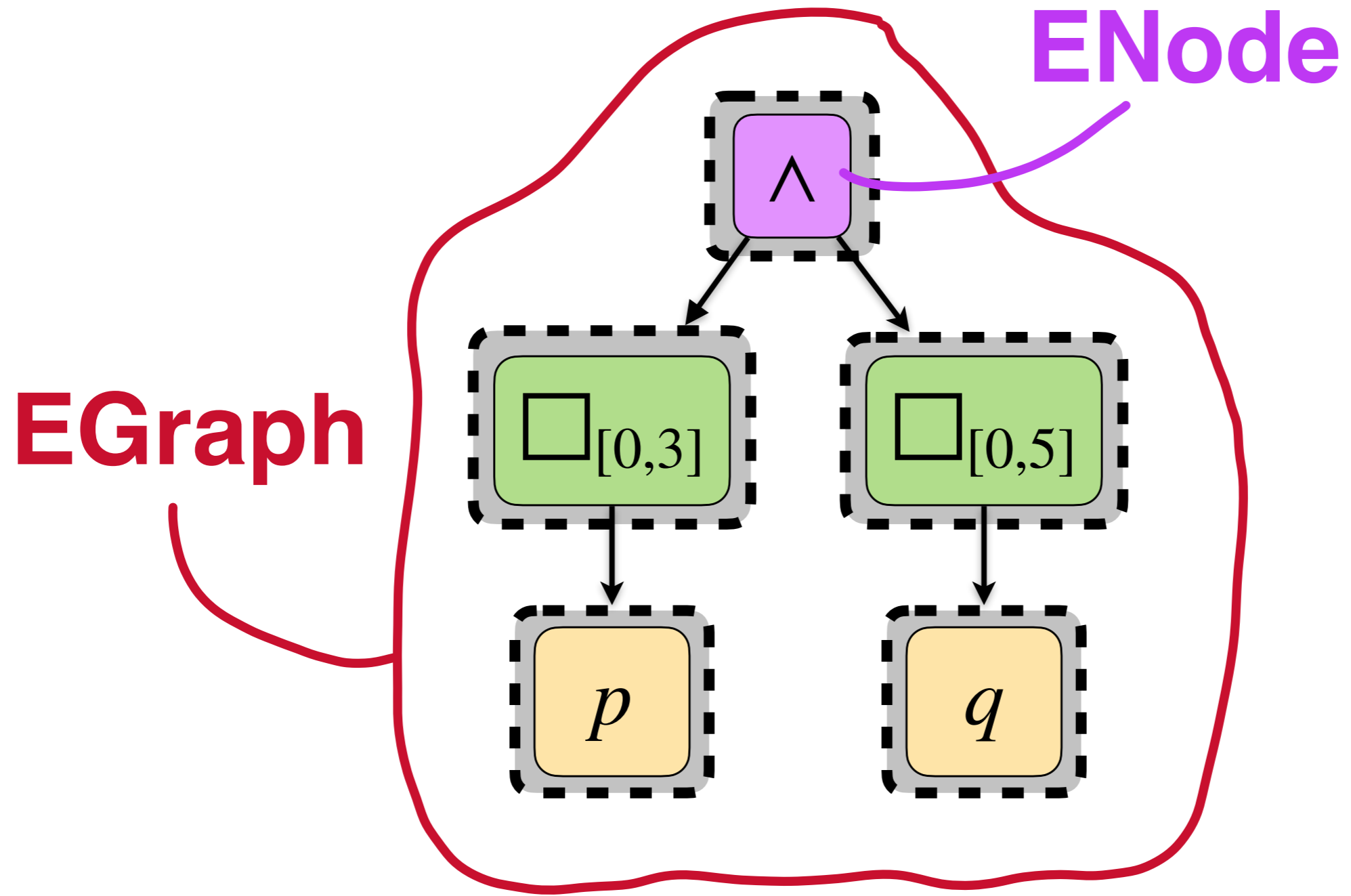
$$\square_{[0,3]} p \wedge \square_{[0,5]} q$$

**EGraph**



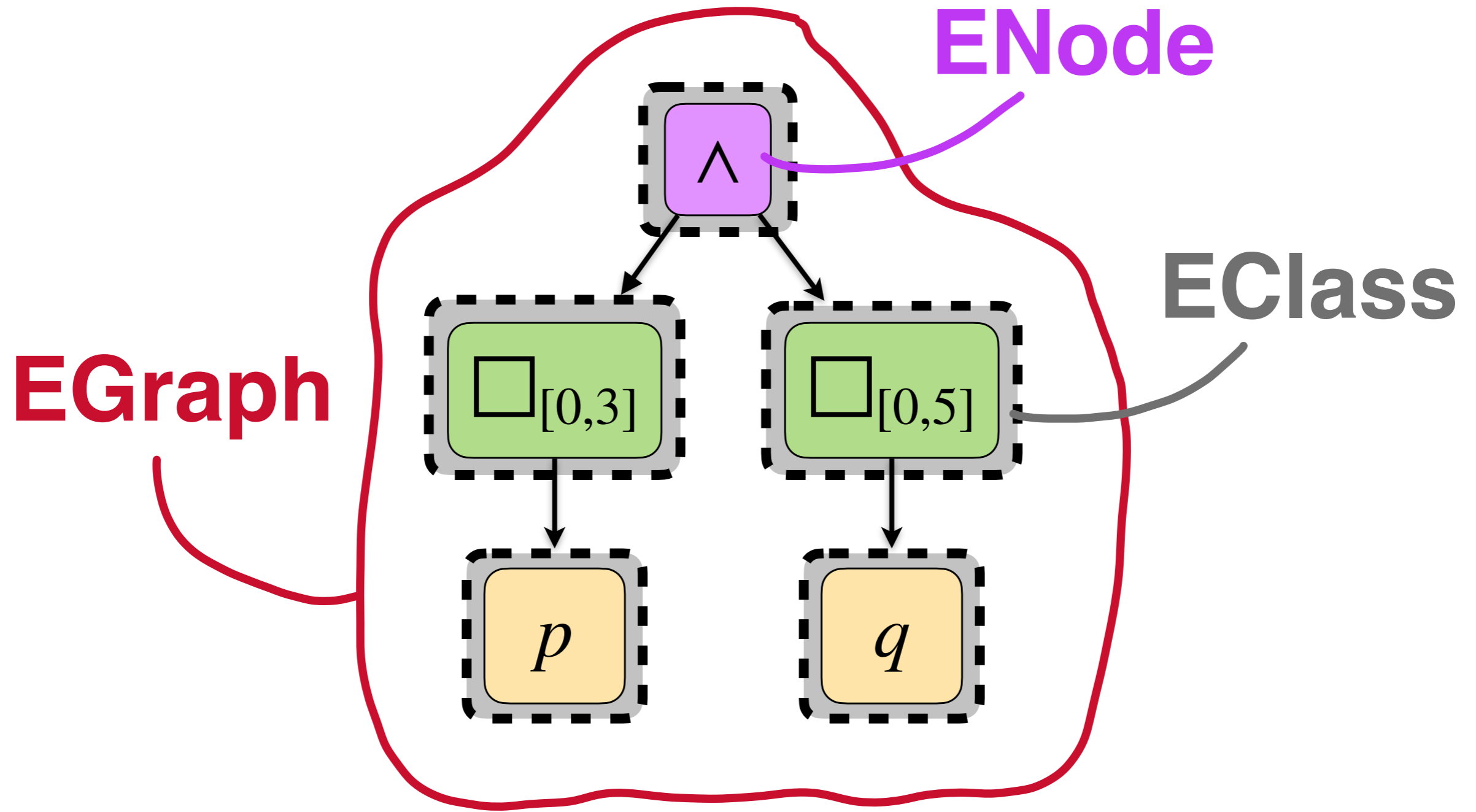
# MLTL Equality Saturation

$$\square_{[0,3]} p \wedge \square_{[0,5]} q$$



# MLTL Equality Saturation

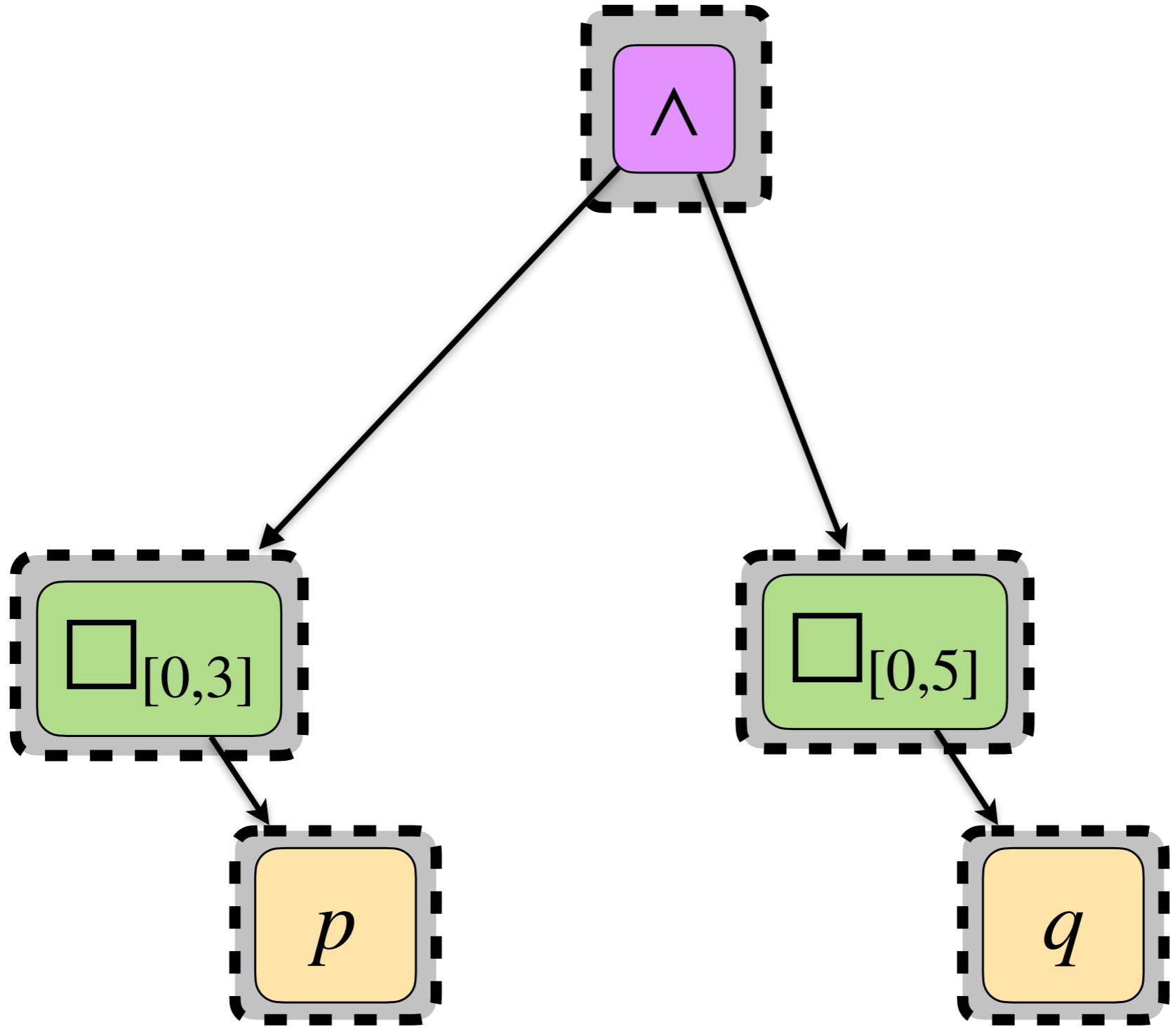
$$\square_{[0,3]} p \wedge \square_{[0,5]} q$$



# MLTL Equality Saturation

$$\square_{[0,3]}p \wedge \square_{[0,5]}q$$

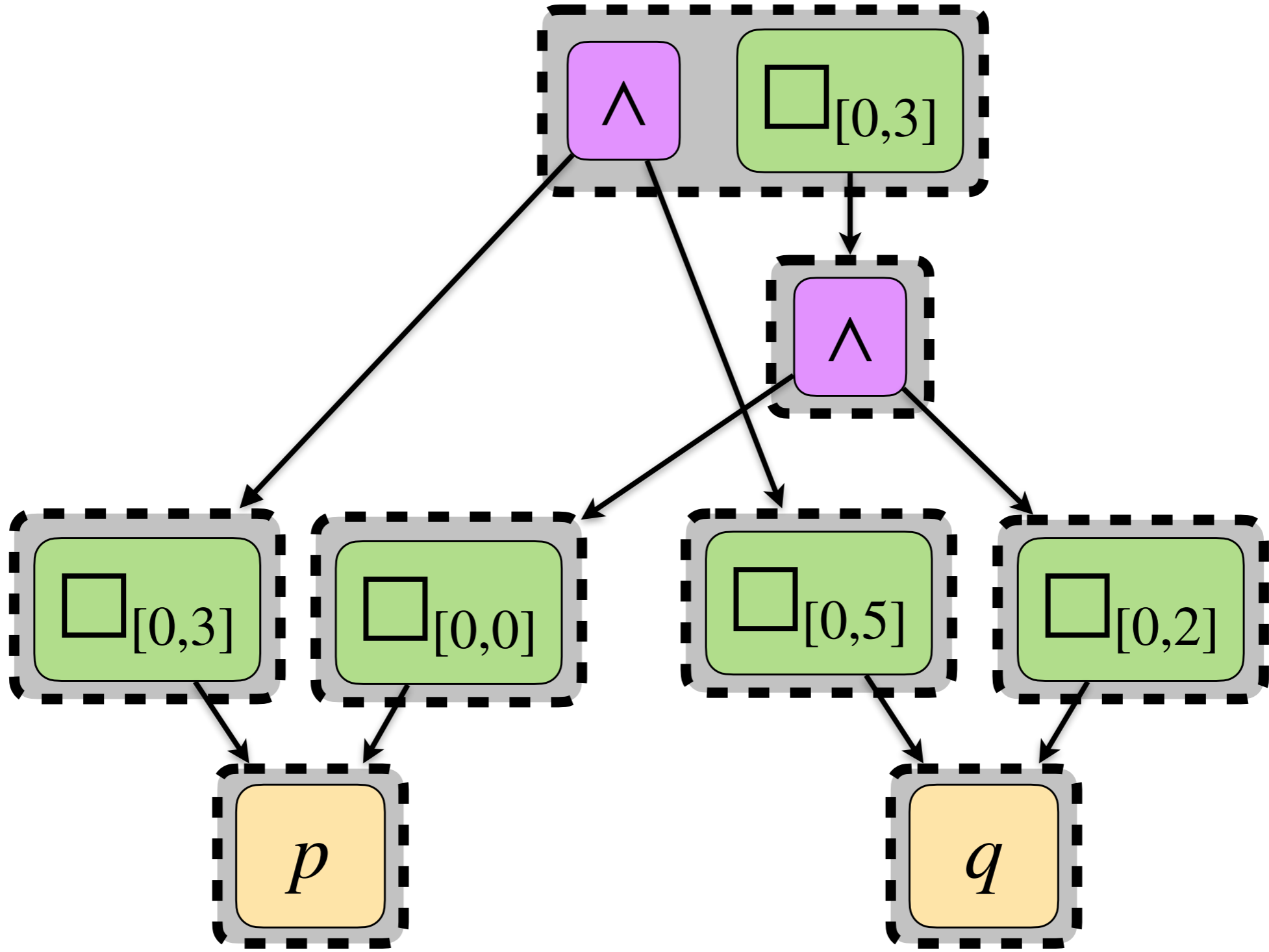
$$\Rightarrow \square_{[0,3]}(\square_{[0,0]}p \wedge \square_{[0,2]}q)$$



# MLTL Equality Saturation

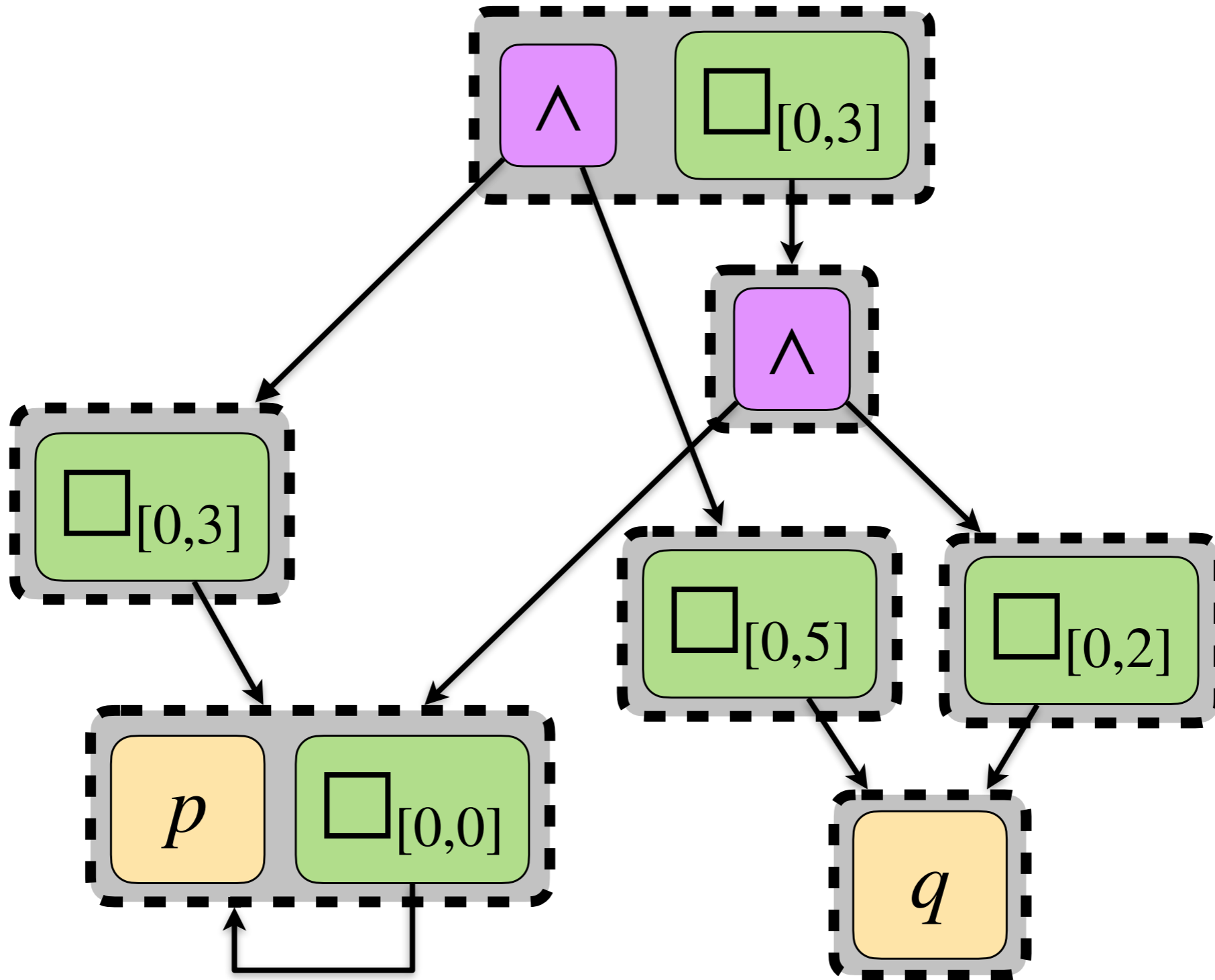
$$\square_{[0,3]}p \wedge \square_{[0,5]}q$$

$$\Rightarrow \square_{[0,3]}(\square_{[0,0]}p \wedge \square_{[0,2]}q)$$



# MLTL Equality Saturation

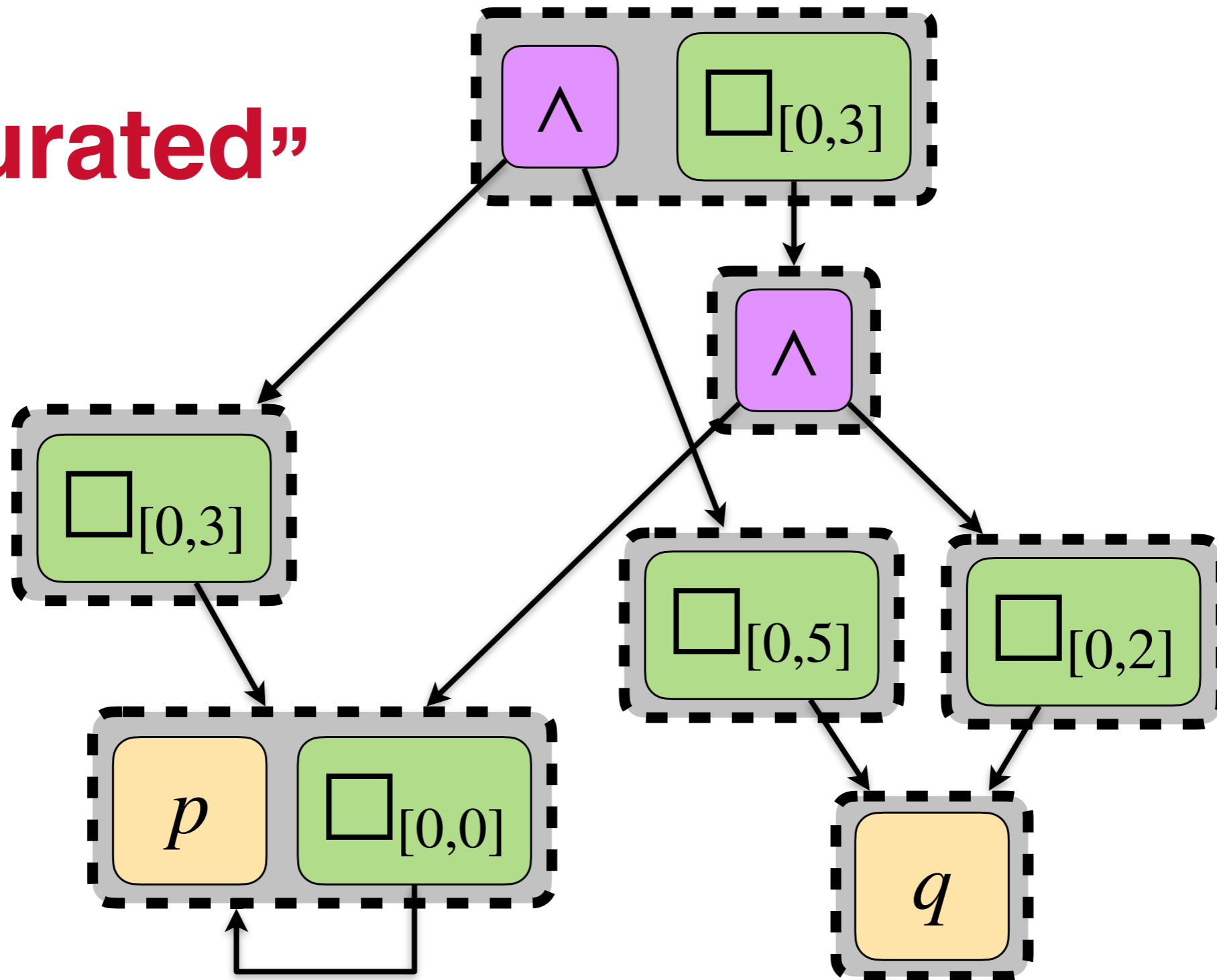
$$\begin{aligned} & \square_{[0,3]} p \wedge \square_{[0,5]} q \\ \Rightarrow & \square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q) \\ \Rightarrow & \square_{[0,3]} (p \wedge \square_{[0,2]} q) \end{aligned}$$



# MLTL Equality Saturation

$$\begin{aligned} & \square_{[0,3]} p \wedge \square_{[0,5]} q \\ \Rightarrow & \square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q) \\ \Rightarrow & \square_{[0,3]} (p \wedge \square_{[0,2]} q) \end{aligned}$$

“Saturated”

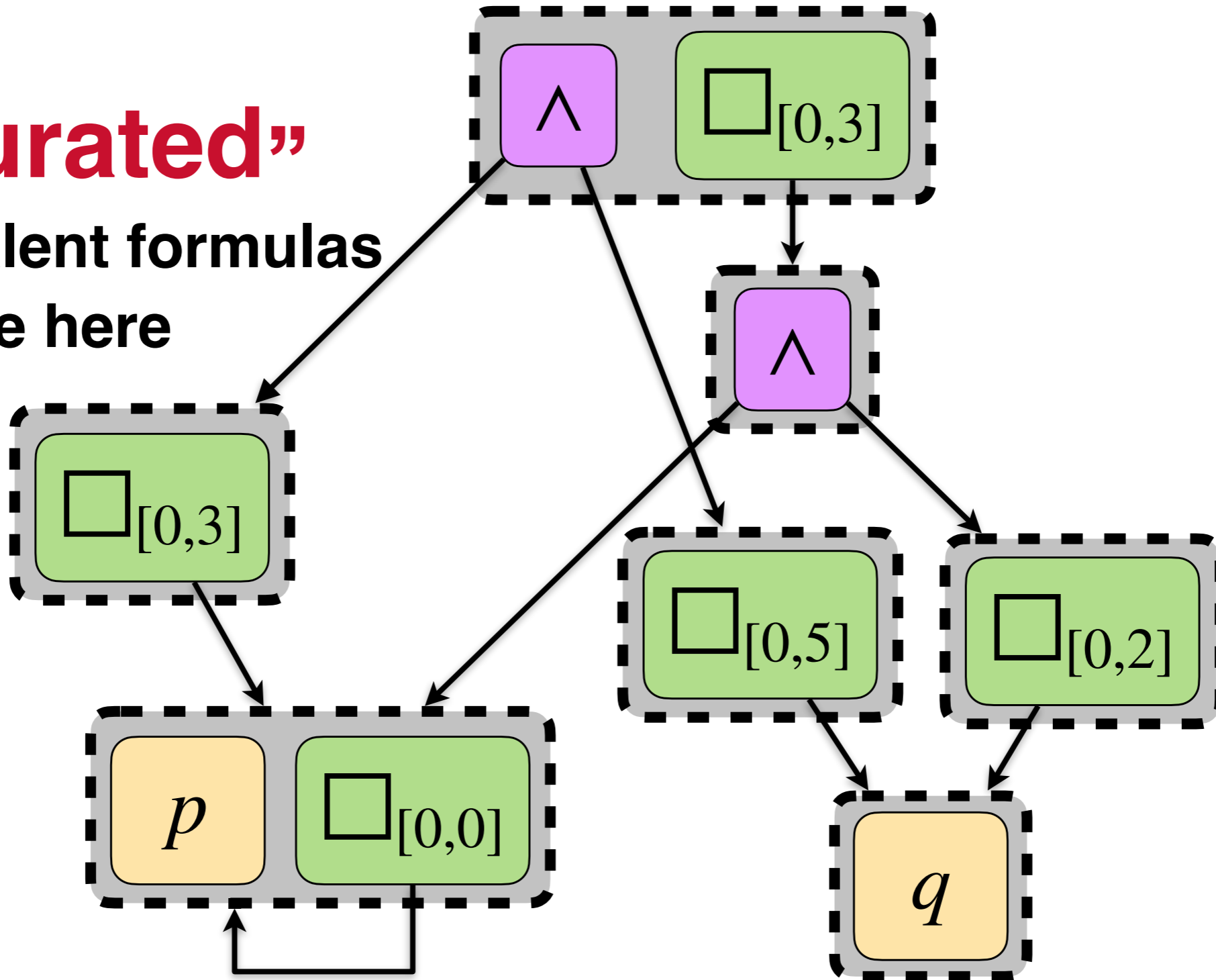


# MLTL Equality Saturation

$$\begin{aligned} & \square_{[0,3]} p \wedge \square_{[0,5]} q \\ \Rightarrow & \square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q) \\ \Rightarrow & \square_{[0,3]} (p \wedge \square_{[0,2]} q) \end{aligned}$$

**“Saturated”**

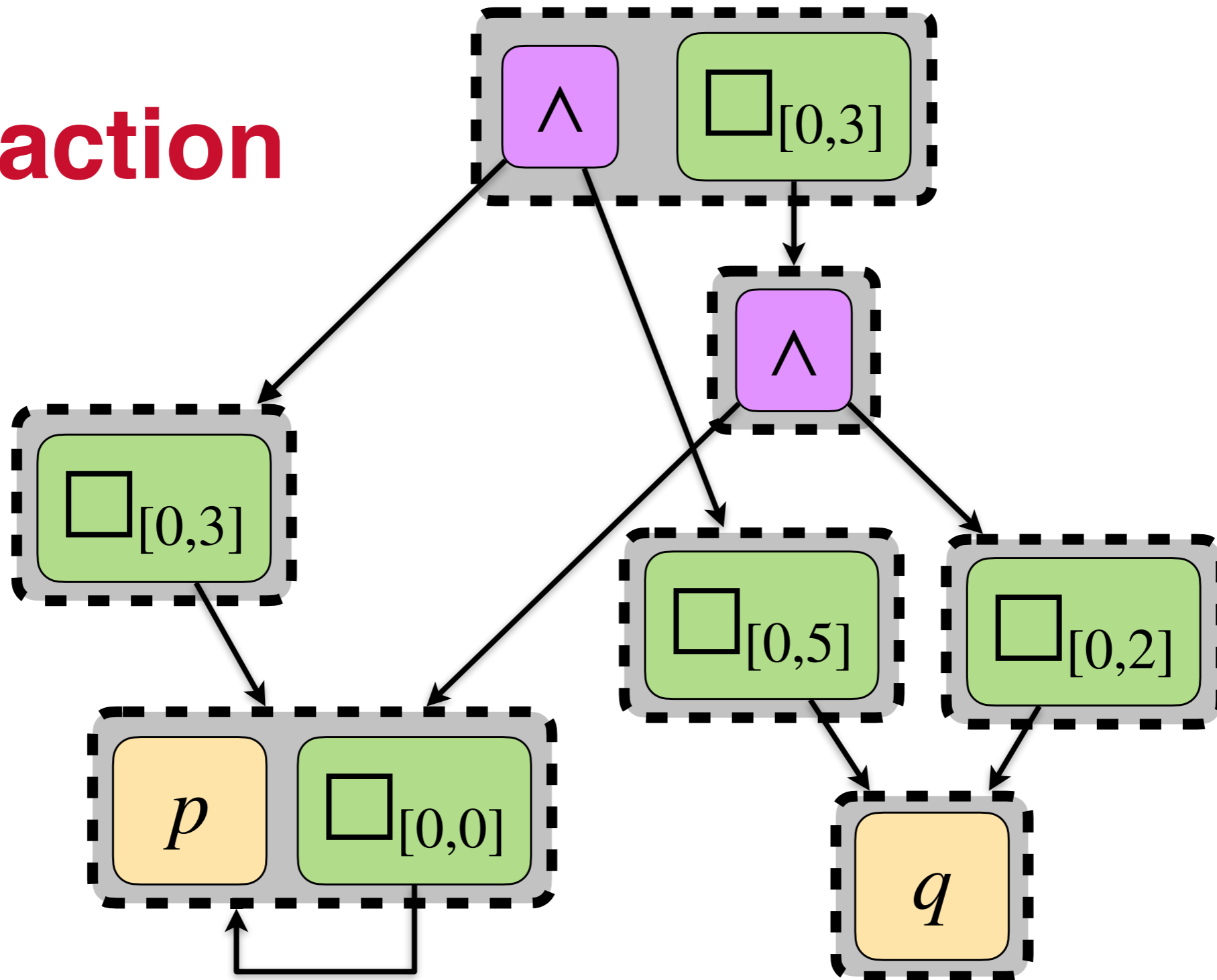
All equivalent formulas to orig. are here



# MLTL Equality Saturation

$$\begin{aligned} & \square_{[0,3]} p \wedge \square_{[0,5]} q \\ \Rightarrow & \square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q) \\ \Rightarrow & \square_{[0,3]} (p \wedge \square_{[0,2]} q) \end{aligned}$$

## Extraction

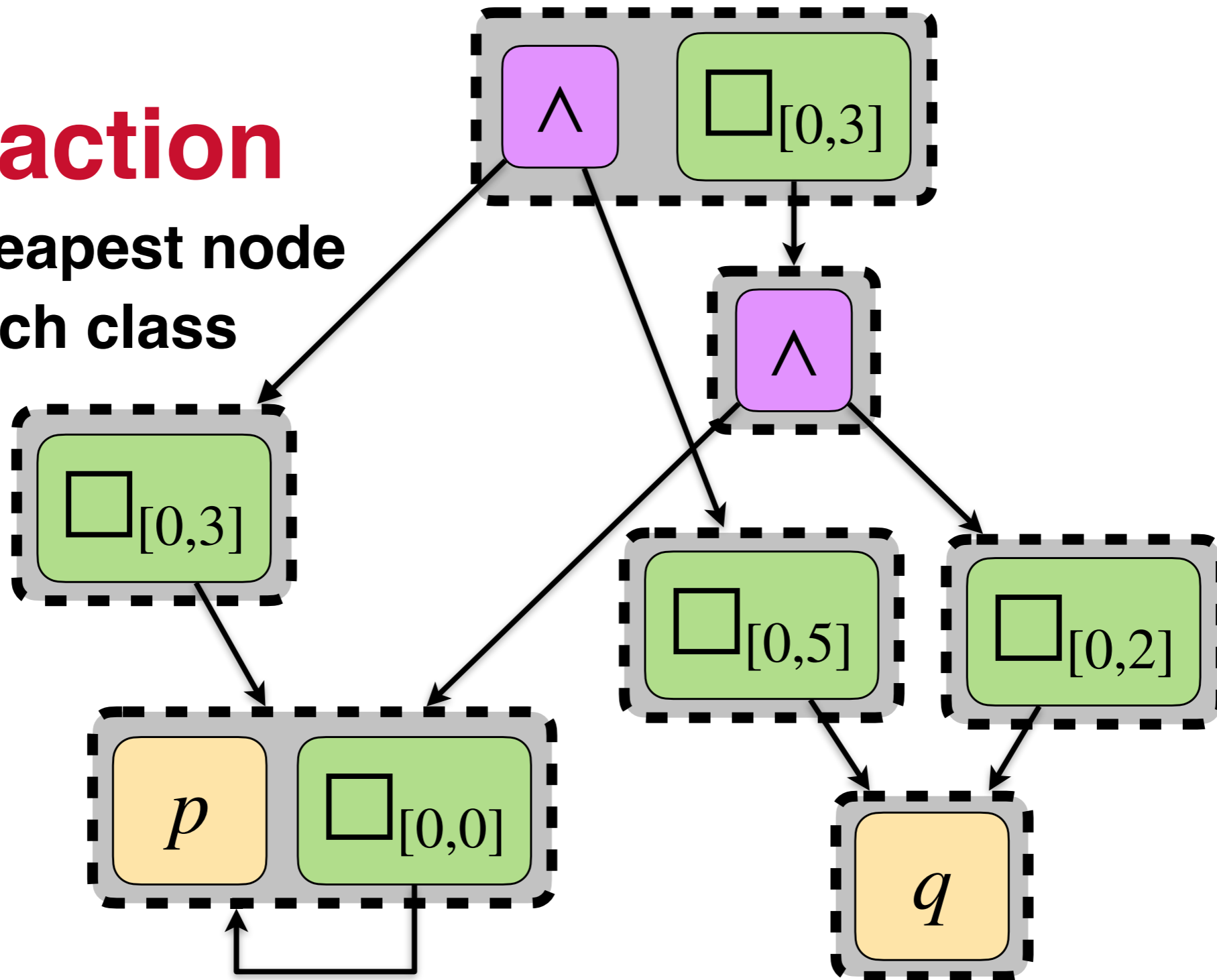


# MLTL Equality Saturation

$$\begin{aligned} & \square_{[0,3]} p \wedge \square_{[0,5]} q \\ \Rightarrow & \square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q) \\ \Rightarrow & \square_{[0,3]} (p \wedge \square_{[0,2]} q) \end{aligned}$$

## Extraction

Pick cheapest node from each class

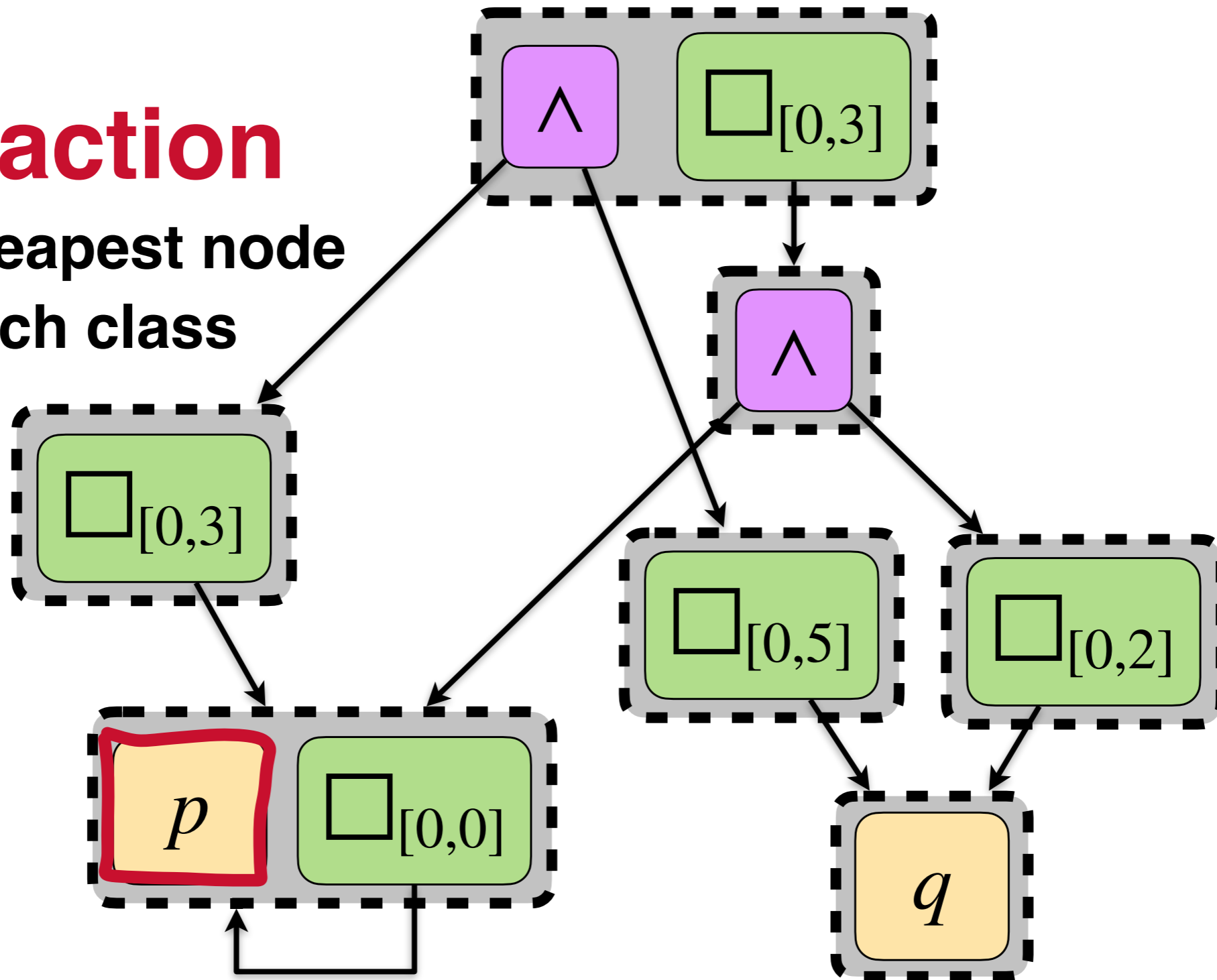


# MLTL Equality Saturation

$$\begin{aligned} & \square_{[0,3]} p \wedge \square_{[0,5]} q \\ \Rightarrow & \square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q) \\ \Rightarrow & \square_{[0,3]} (p \wedge \square_{[0,2]} q) \end{aligned}$$

## Extraction

Pick cheapest node from each class

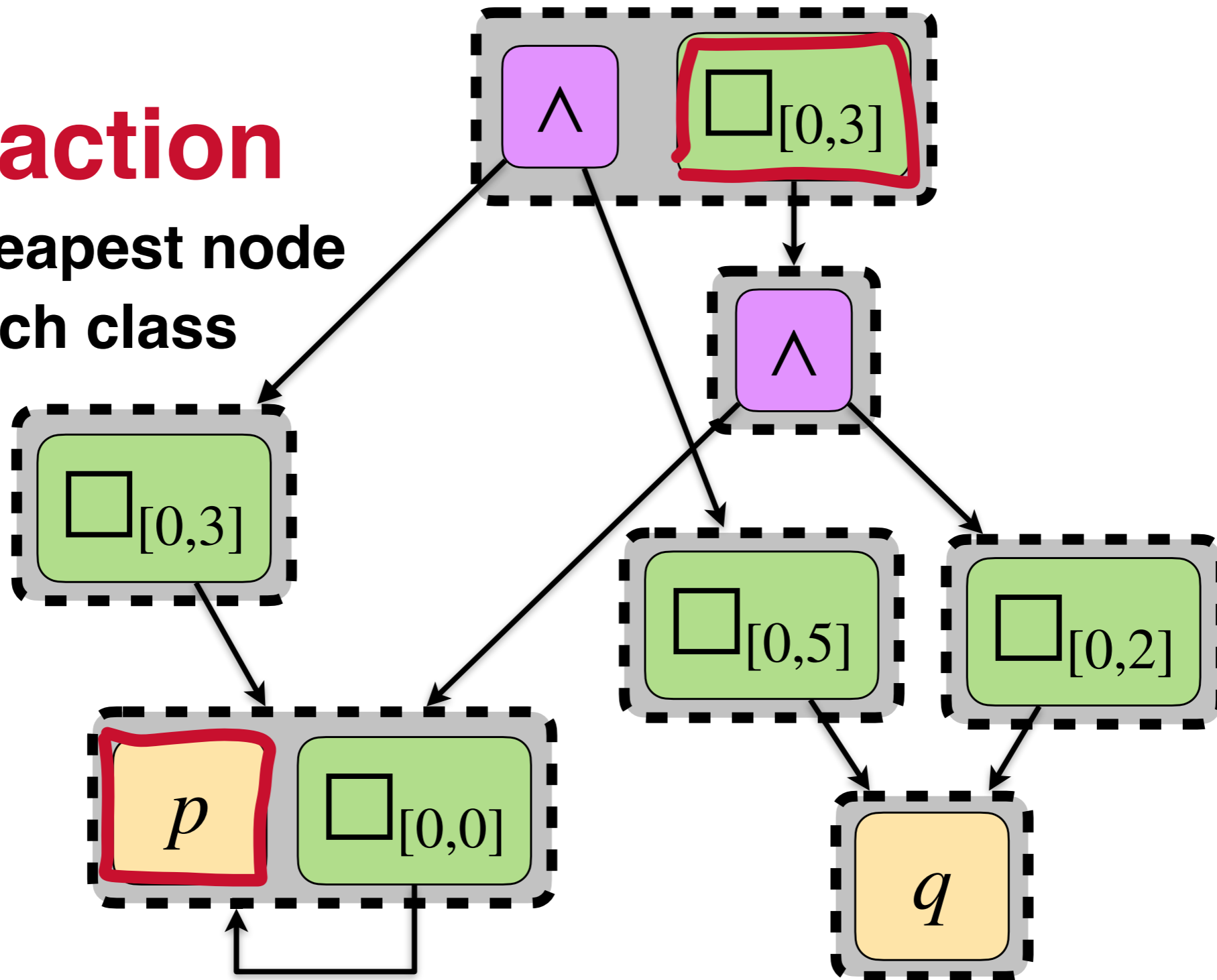


# MLTL Equality Saturation

$$\begin{aligned} & \square_{[0,3]} p \wedge \square_{[0,5]} q \\ \Rightarrow & \square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q) \\ \Rightarrow & \square_{[0,3]} (p \wedge \square_{[0,2]} q) \end{aligned}$$

## Extraction

Pick cheapest node from each class

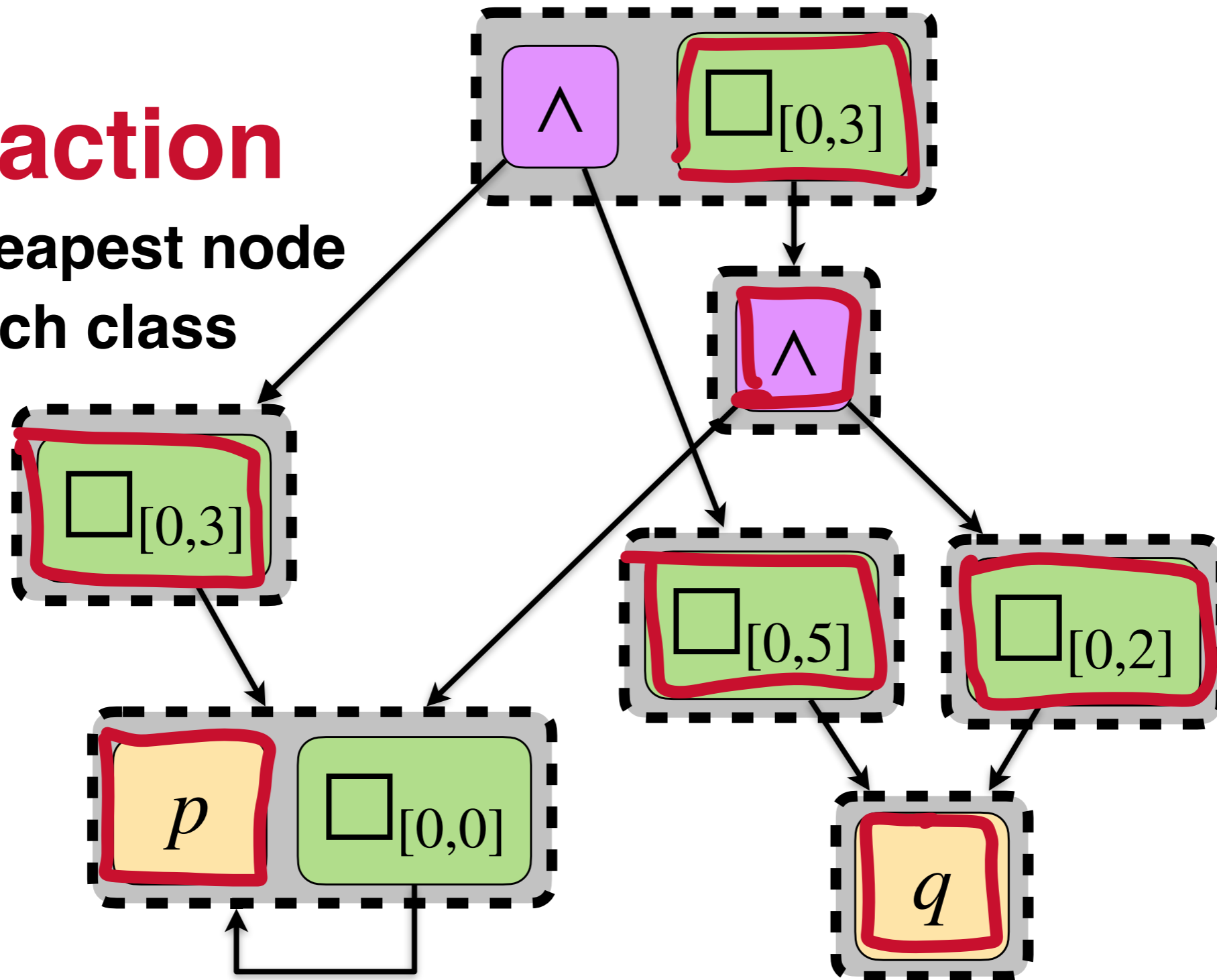


# MLTL Equality Saturation

$$\begin{aligned} & \square_{[0,3]} p \wedge \square_{[0,5]} q \\ \Rightarrow & \square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q) \\ \Rightarrow & \square_{[0,3]} (p \wedge \square_{[0,2]} q) \end{aligned}$$

## Extraction

Pick cheapest node from each class

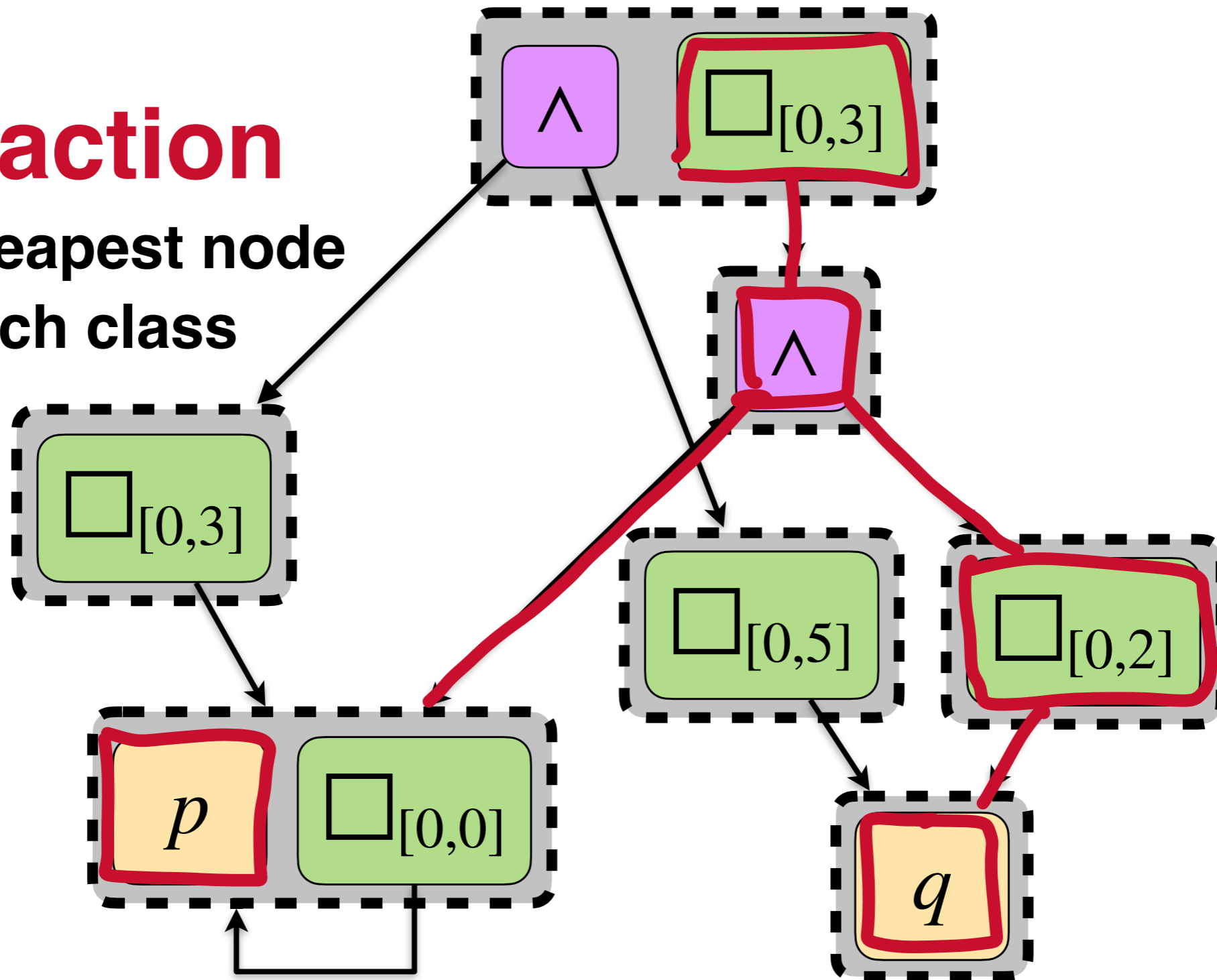


# MLTL Equality Saturation

$$\begin{aligned} & \square_{[0,3]} p \wedge \square_{[0,5]} q \\ \Rightarrow & \square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q) \\ \Rightarrow & \square_{[0,3]} (p \wedge \square_{[0,2]} q) \end{aligned}$$

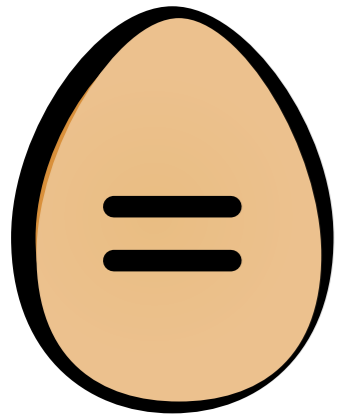
## Extraction

Pick cheapest node from each class

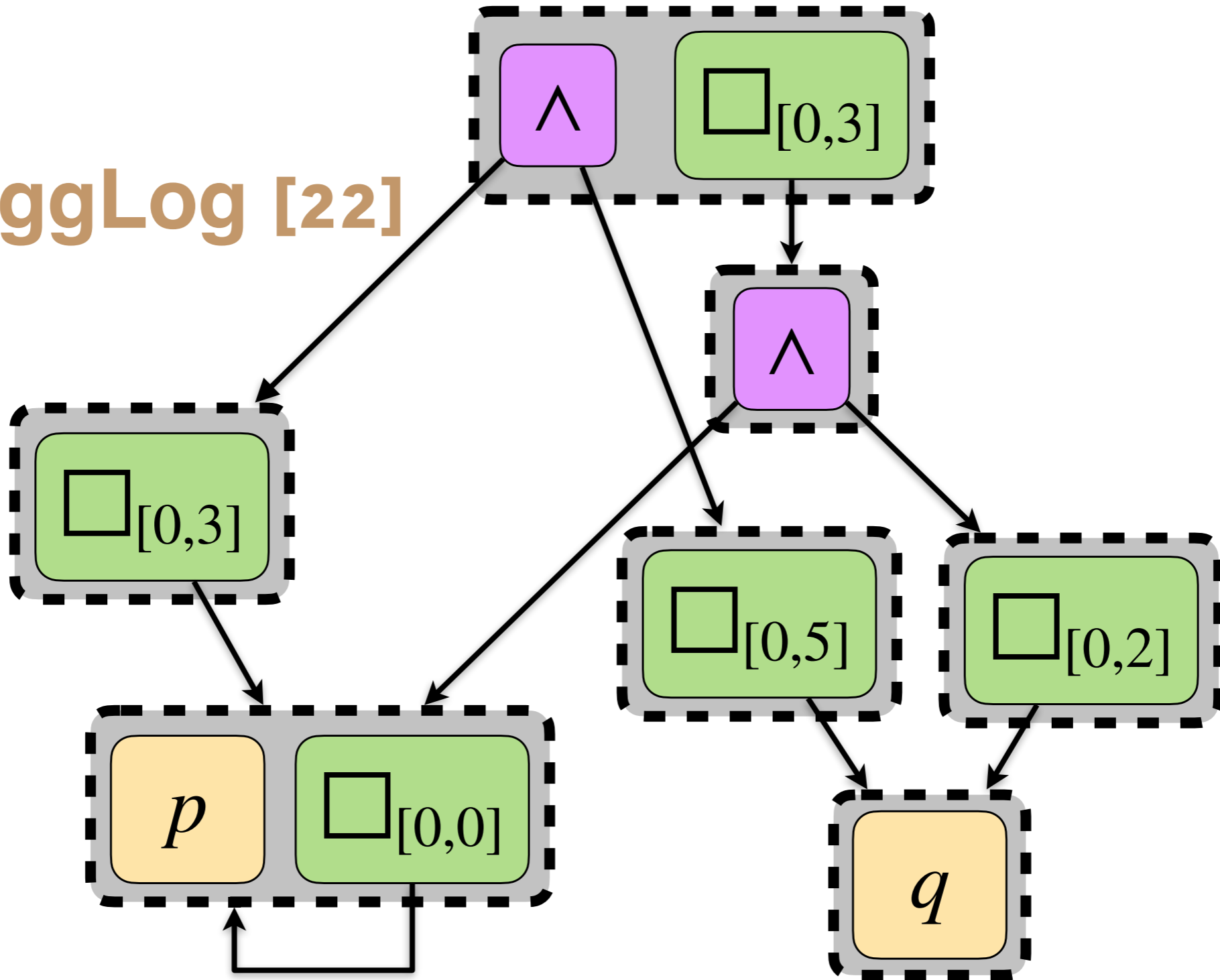


# MLTL Equality Saturation

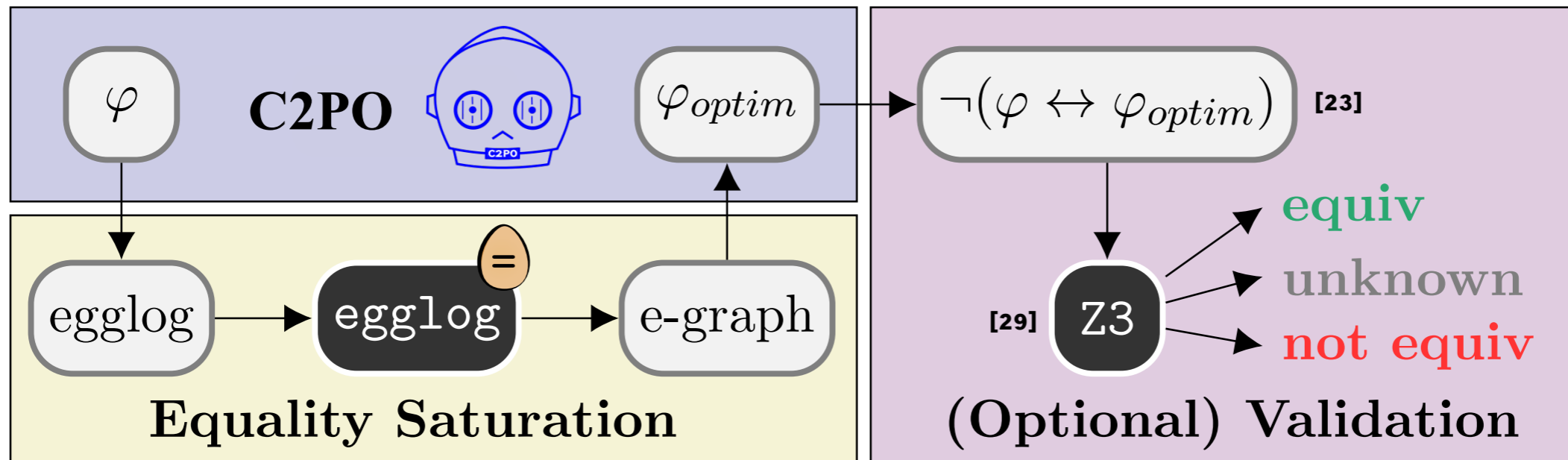
$$\begin{aligned} & \square_{[0,3]} p \wedge \square_{[0,5]} q \\ \Rightarrow & \square_{[0,3]} (\square_{[0,0]} p \wedge \square_{[0,2]} q) \\ \Rightarrow & \square_{[0,3]} (p \wedge \square_{[0,2]} q) \end{aligned}$$



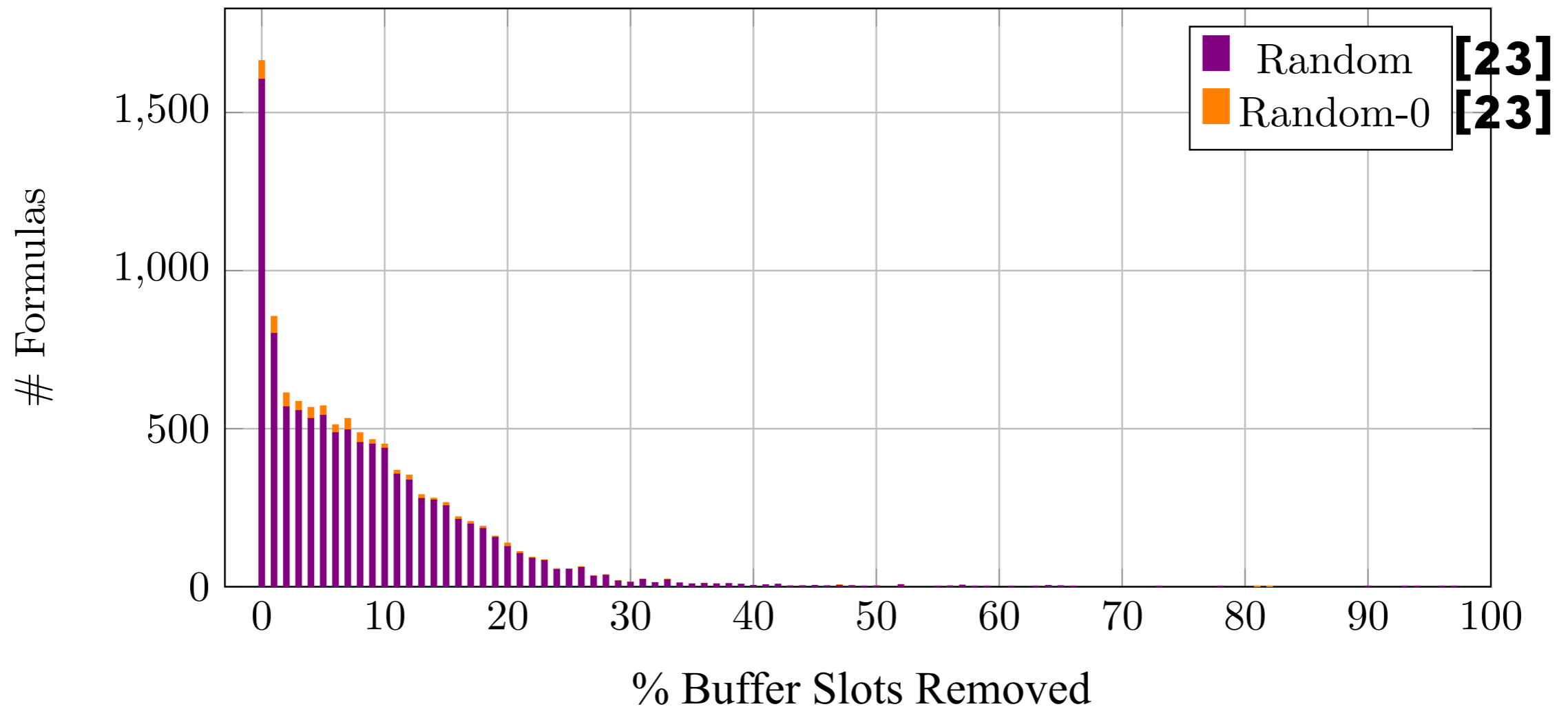
EggLog [22]



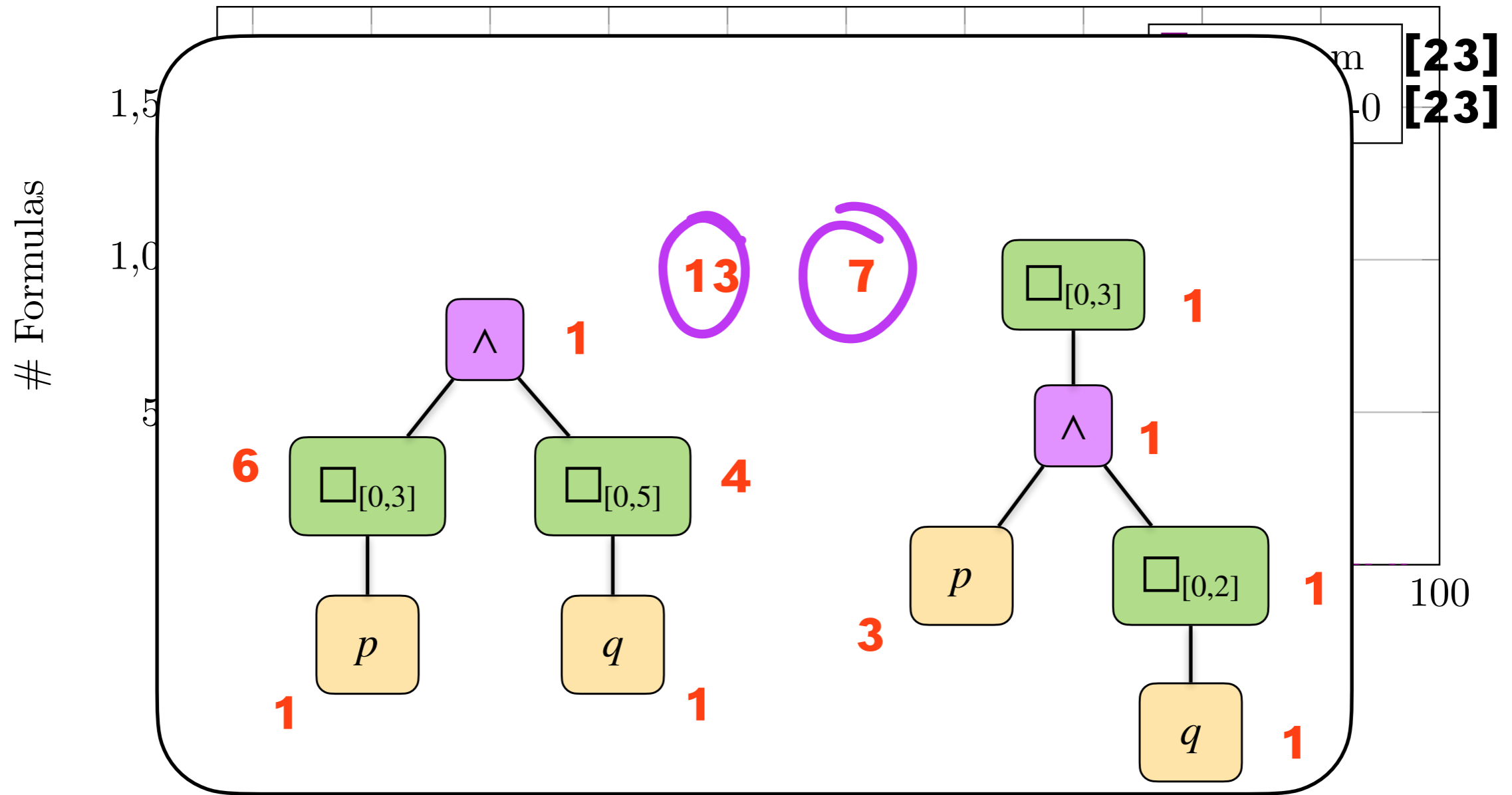
# MLTL Equality Saturation Implementation



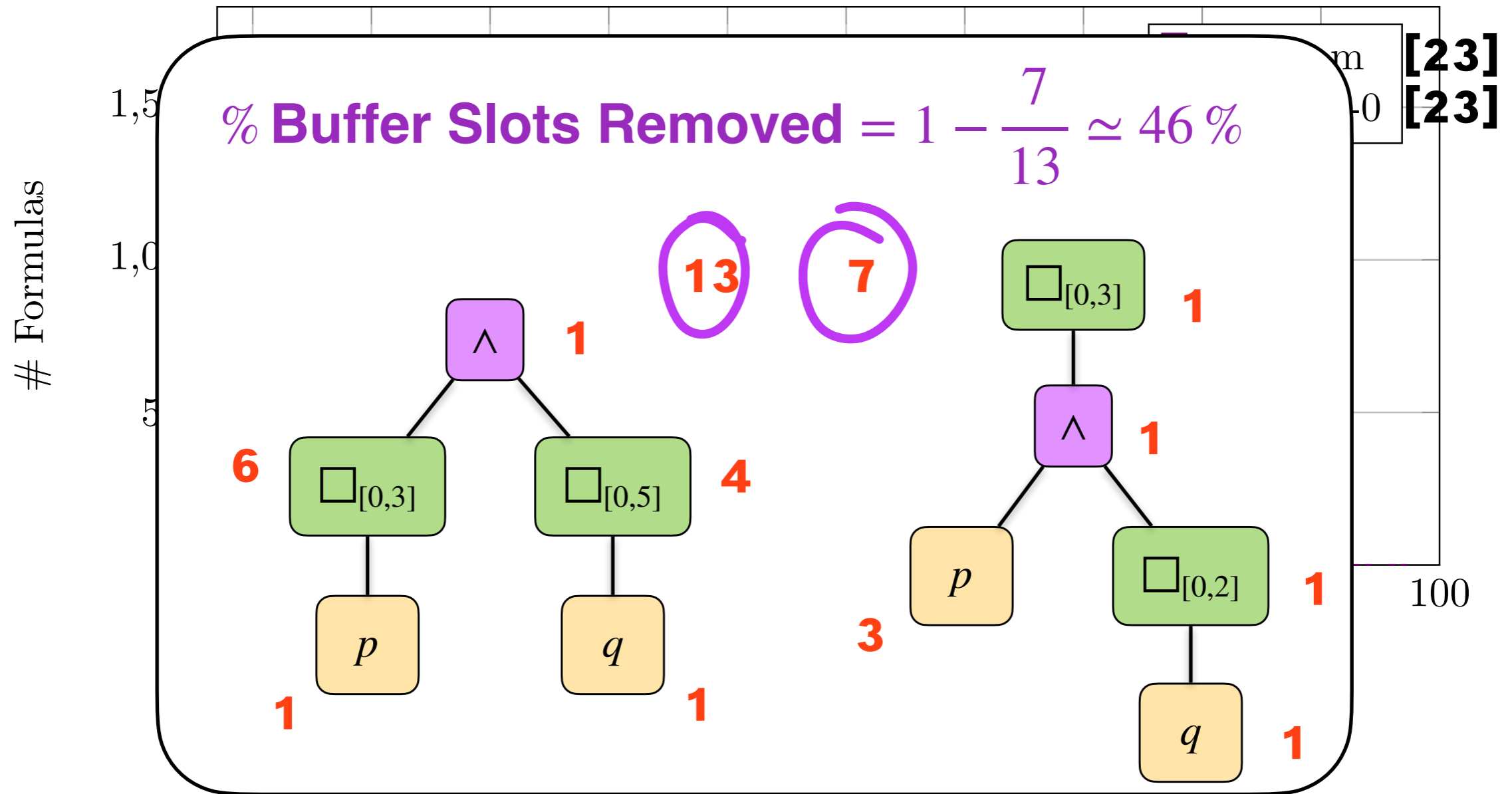
# MLTL Equality Saturation Results (Random)



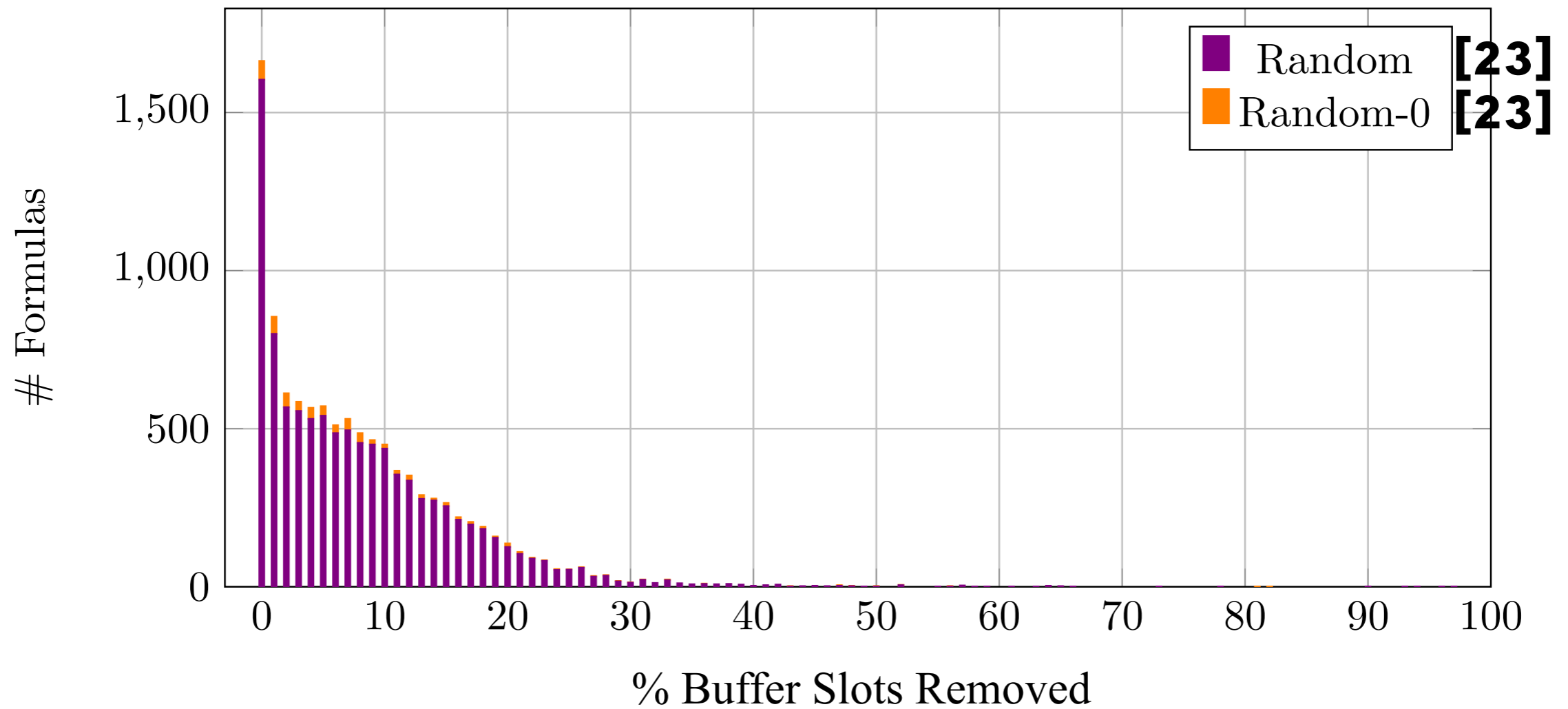
# MLTL Equality Saturation Results (Random)



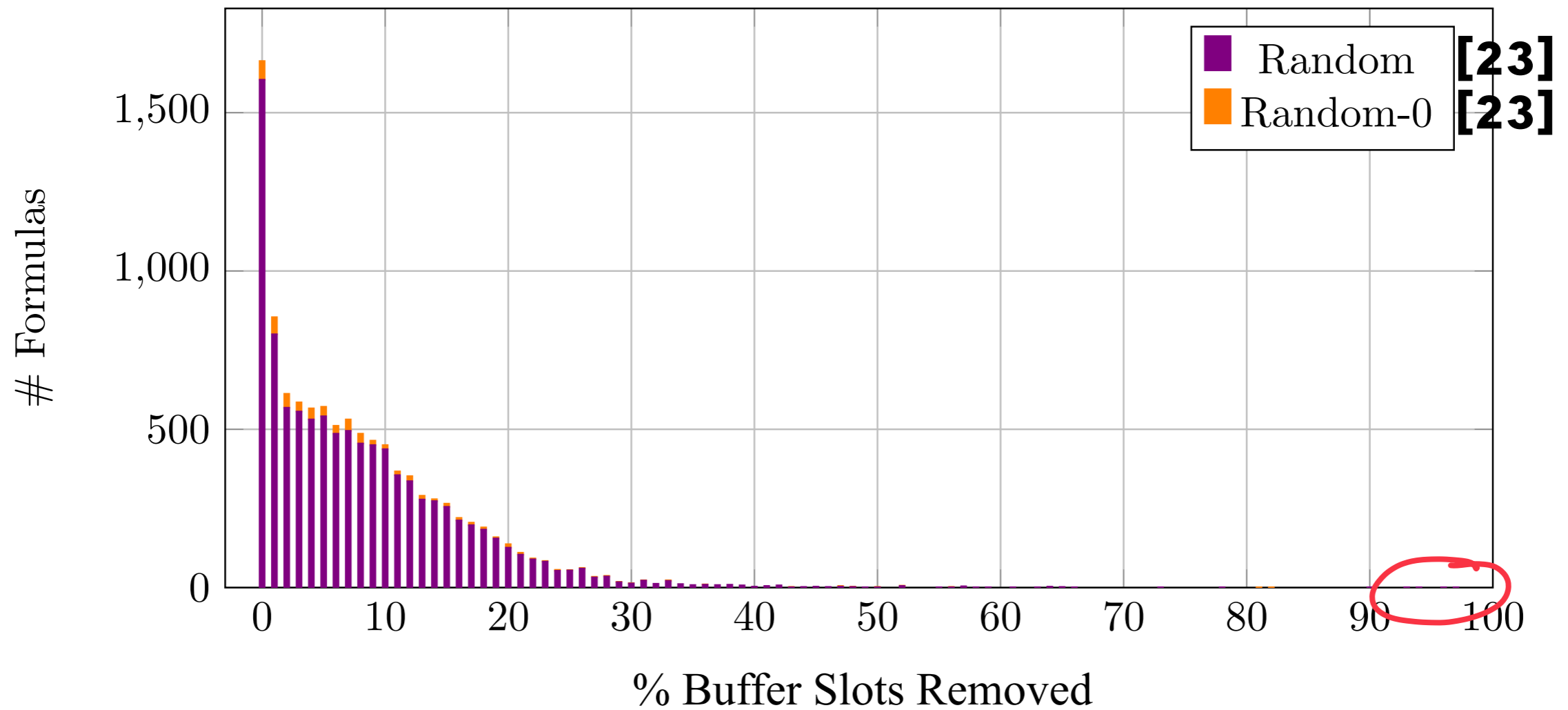
# MLTL Equality Saturation Results (Random)



# MLTL Equality Saturation Results (Random)



# MLTL Equality Saturation Results (Random)



# MLTL Equality Saturation Results (Human Authored)



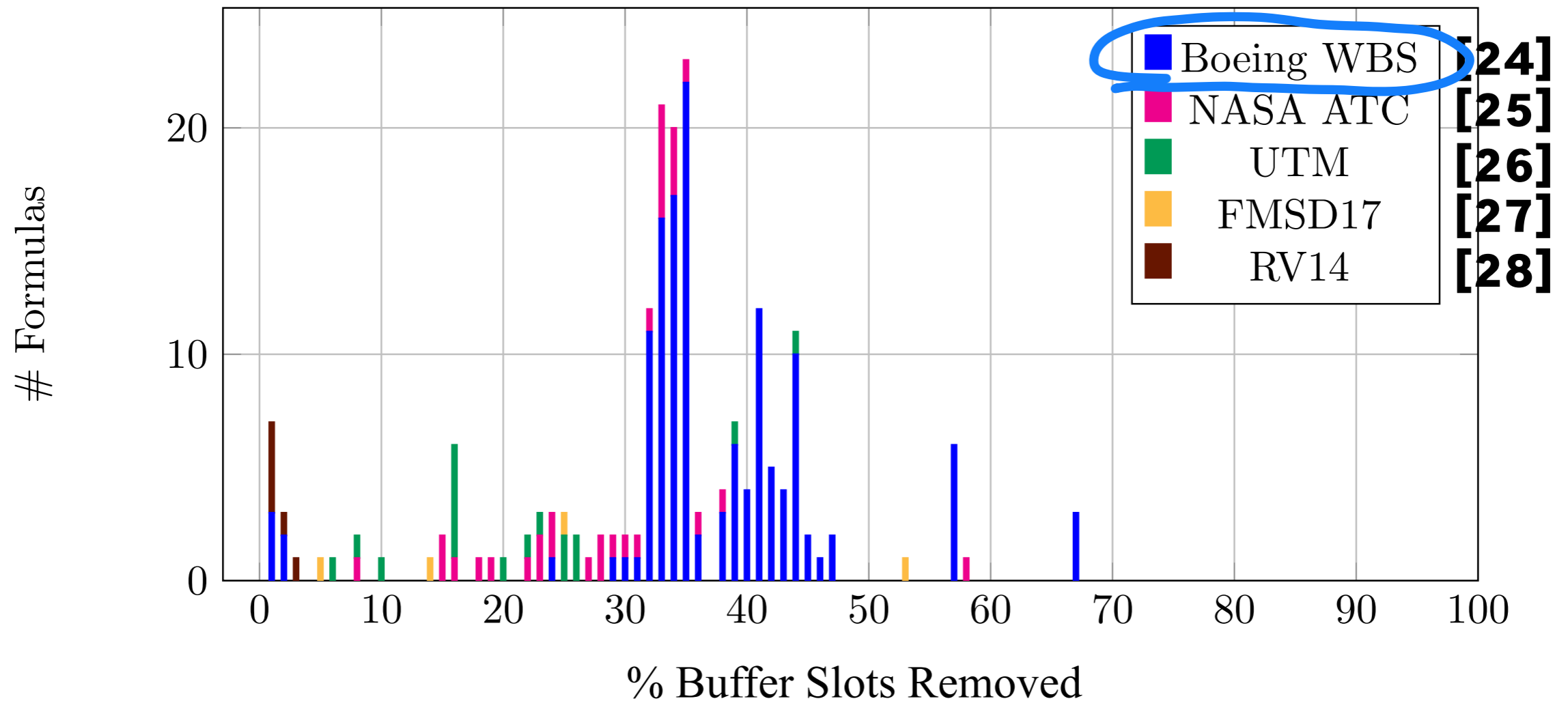
# MLTL Equality Saturation Results (Human Authored)



\*\*\* < 1% reduction omitted

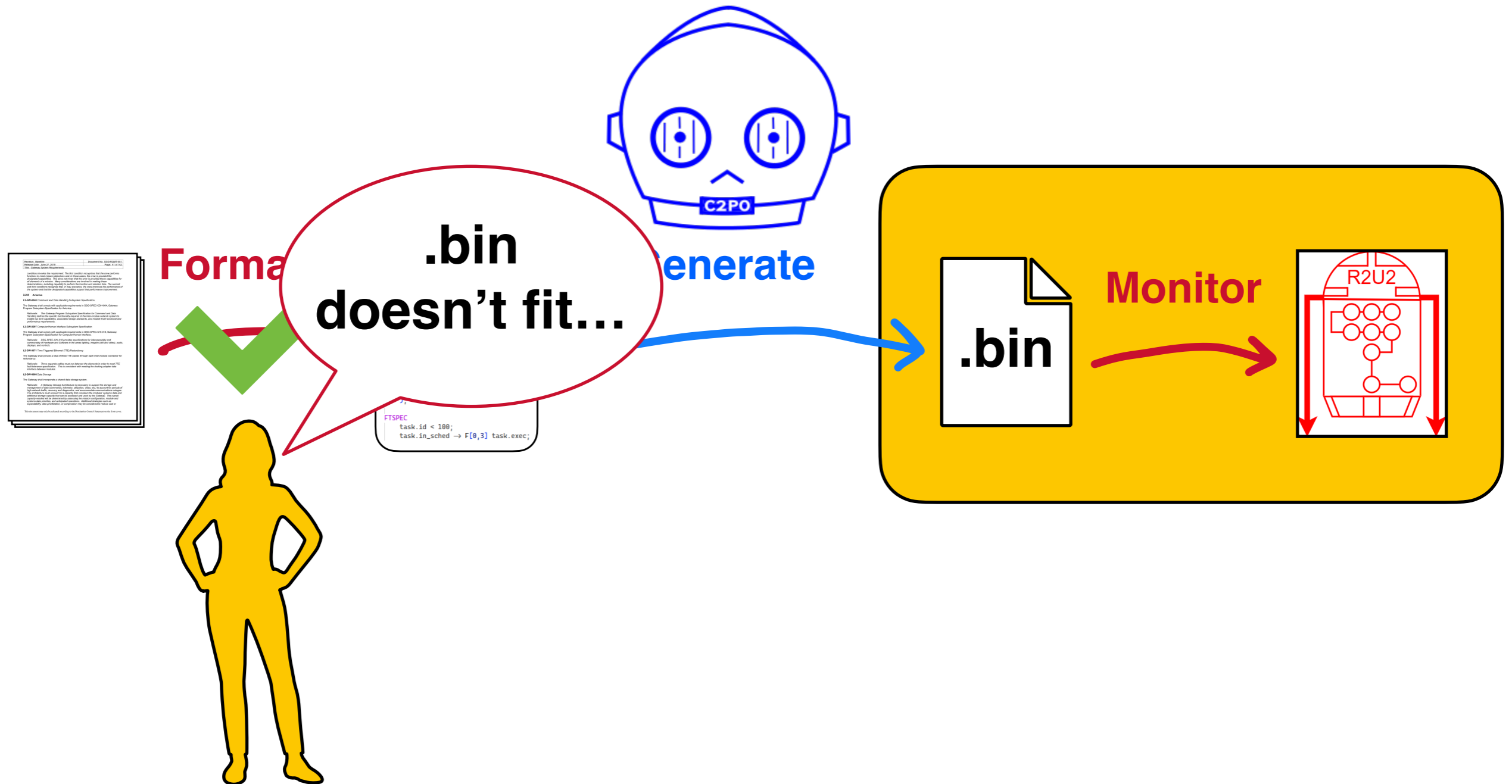
# MLTL Equality Saturation Results (Human Authored)

~35% Average Reduction

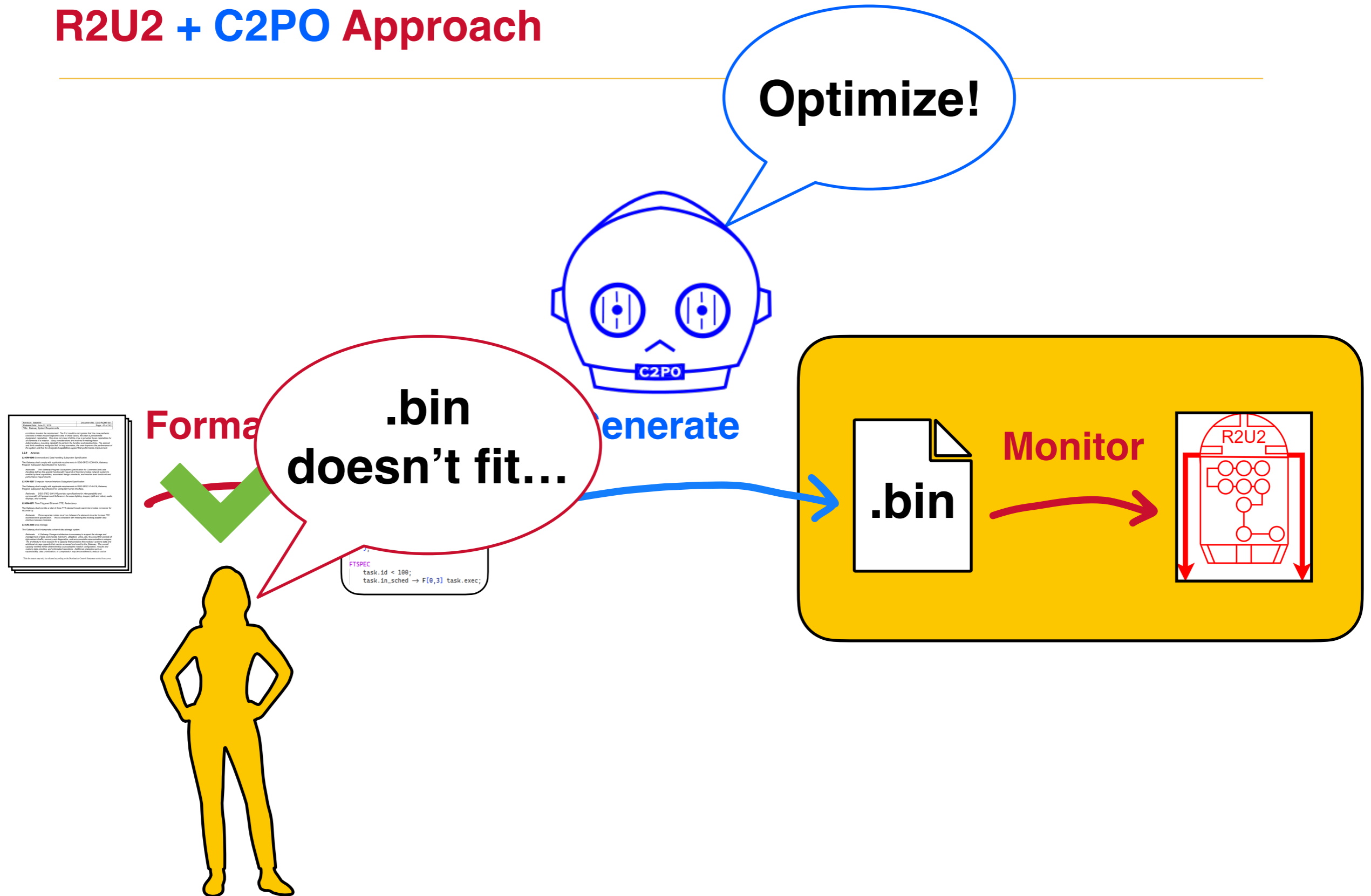


\*\*\* < 1% reduction omitted

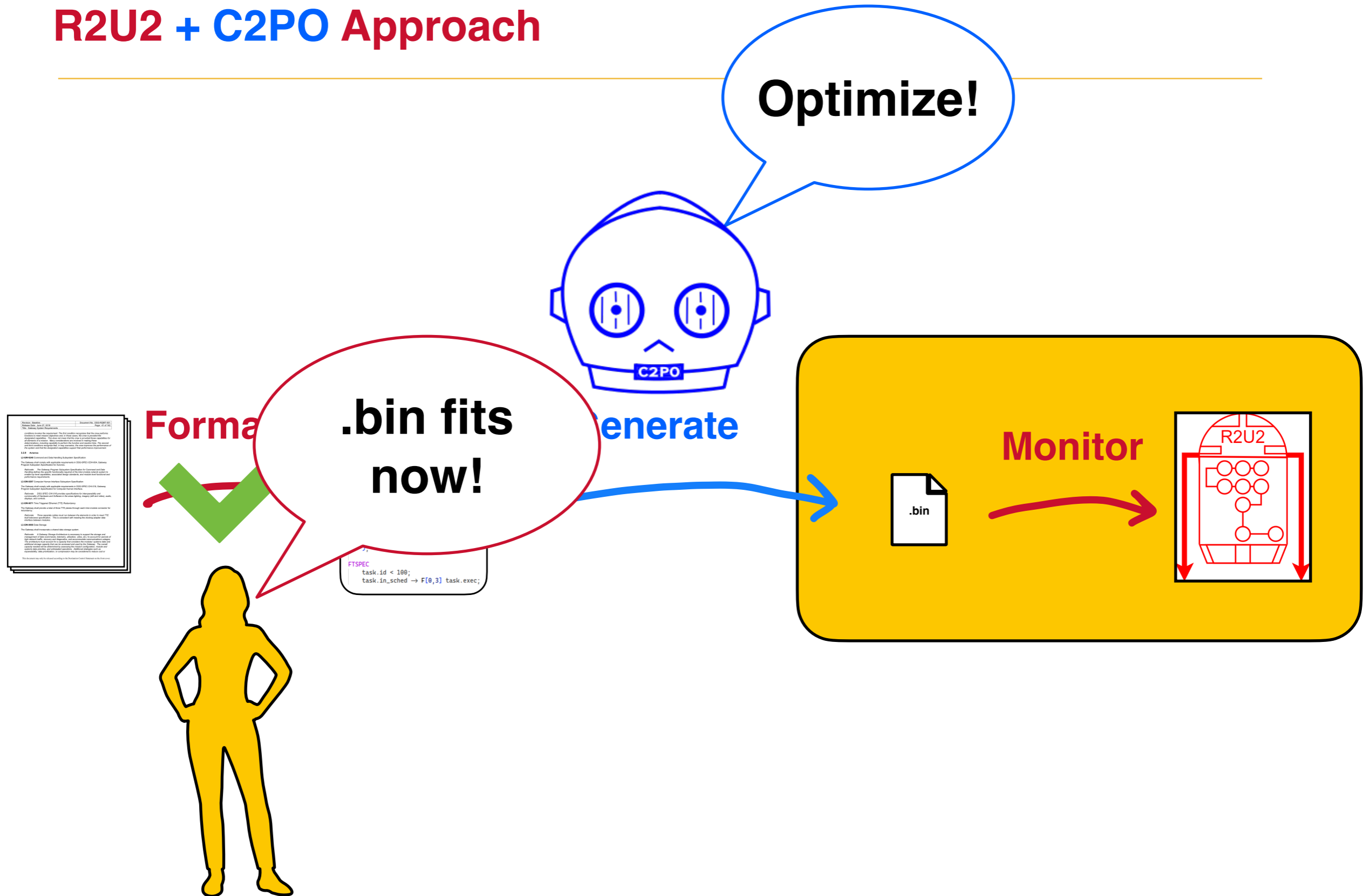
# R2U2 + C2PO Approach



# R2U2 + C2PO Approach



# R2U2 + C2PO Approach



# R2U2 + C2PO Approach

You're welcome!



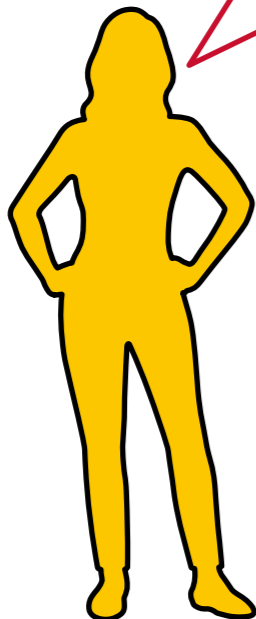
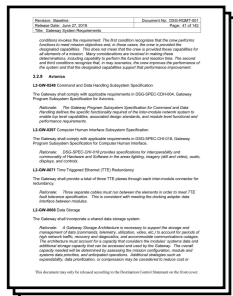
Format

Thanks,  
C2PO!


generate



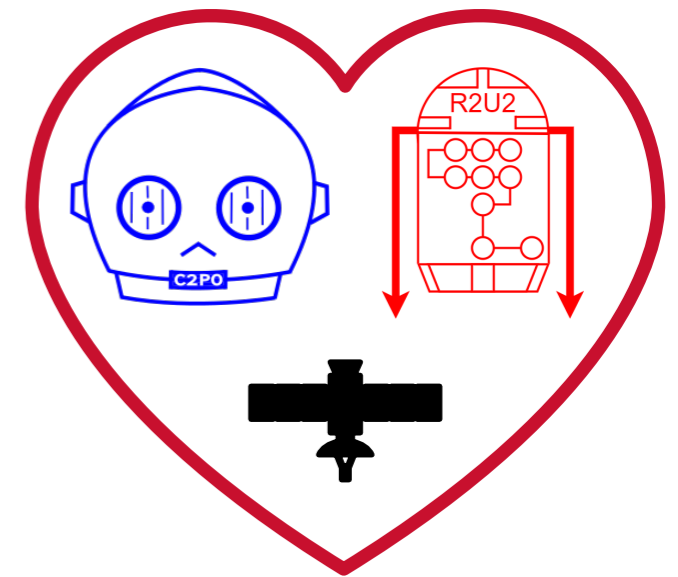
```
FTSPEC
task.id < 100;
task.in_sched → F[0,3] task.exec;
```



# Questions?

- > **C2PO**
  - > Optimized encoding of MLTL monitors for R2U2
- > **FO-MLTL**
  - > First-order MLTL over bounded dynamic sets
- > **MLTL Optimizations**
  - > Rewrite Rules 
  - > Equality saturation via EggLog
- > **Future Directions**
  - > Implement FO-MLTL in R2U2
  - > More rewrites!

[r2u2.temporallogic.org](http://r2u2.temporallogic.org)



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